C2 - Chapter 11 - Integration - Summary

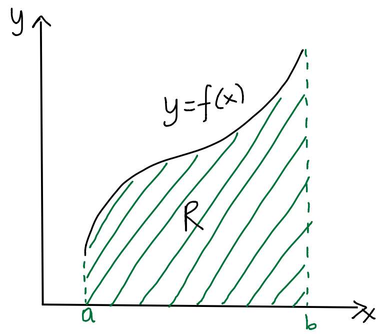
* $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ where c is an arbitrary constant (also known as a constant of integration).

The above is an indefinite integral since we can only integrate up to a constant.

* $\int_1^2 x^2 \, dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = 7/3$

limits

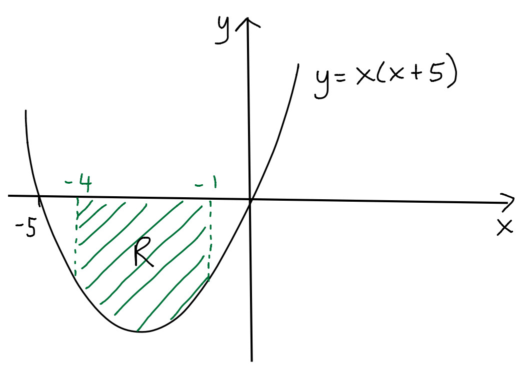
The above is a definite integral (there is no c and you get a numerical answer).



The area enclosed by the curve $y=f(x)$, the x-axis and the lines $x=a$ and $x=b$ is given by

$$\int_a^b f(x) \, dx$$

* Note: We know that area is a positive quantity. If however the region enclosed by the curve, the x-axis and the lines $x=a$ and $x=b$ is below the x-axis, then the integral will give a negative value.



$$R = \int_{-4}^{-1} x(x+5) \, dx = \int_{-4}^{-1} x^2 + 5x \, dx$$

$$= \left[\frac{x^3}{3} + \frac{5x^2}{2}\right]_{-4}^{-1}$$

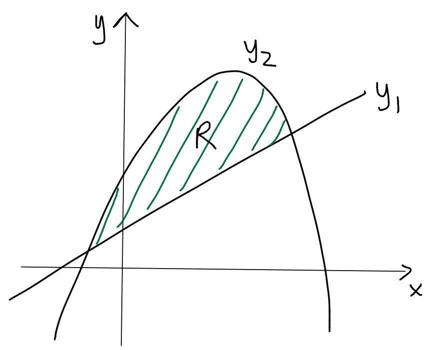
$$= \left(\frac{(-1)^3}{3} + \frac{5(-1)^2}{2}\right) - \left(\frac{(-4)^3}{3} + \frac{5(-4)^2}{2}\right)$$

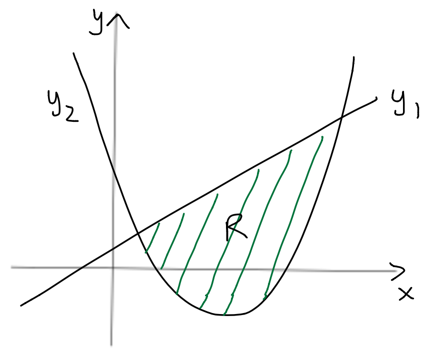
$= -16.5$ So, the area is 16.5 units^2

* Area between a line and a curve

$$\text{Area} = \int_a^b (y_1 - y_2)\,dx$$

where y_1 is the equation of the line (or curve) above and y_2 is the equation of the curve (or line below).



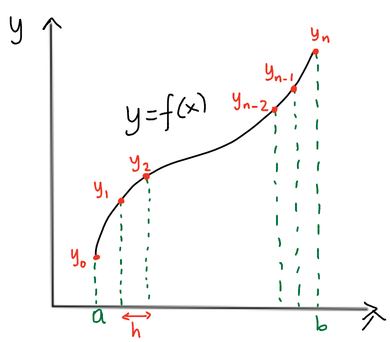


$$R = \int_a^b (y_2 - y_1)\,dx \qquad\qquad R = \int_a^b (y_1 - y_2)\,dx$$

* Remember that you can also break up the required region into different shapes (eg triangles, rectangles).

* Trapezium rule

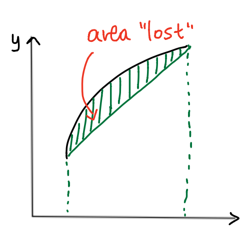
In cases where an integral is not available analytically, we may use the trapezium rule to find an approximation to it.



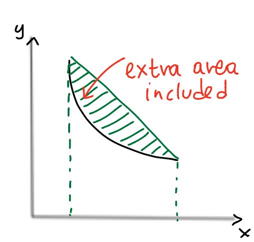
$$\int_a^b f(x)\,dx = \tfrac{1}{2}\cdot h\left\{ y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n \right\}$$

where $h = \dfrac{b-a}{n}$

*



When the curve bends outwards the trapezium rule underestimates the true area.



When the curve bends inwards the trapezium rule overestimates the true area.

* $\% \text{ error} = \dfrac{|\text{exact} - \text{estimate}|}{\text{exact}} \times 100\,\%$