C2 - Chapter 4 - Coordinate geometry in the (x,y) plane - Extra Practice

1. a) Radius $=\sqrt{(4-1)^2+[2-(-2)]^2}=5 \Rightarrow (x-1)^2+(y+2)^2=5^2$

   b) Radius $=\sqrt{(-5-0)^2+(7-5)^2}=\sqrt{29} \Rightarrow (x+5)^2+(y-7)^2=29$

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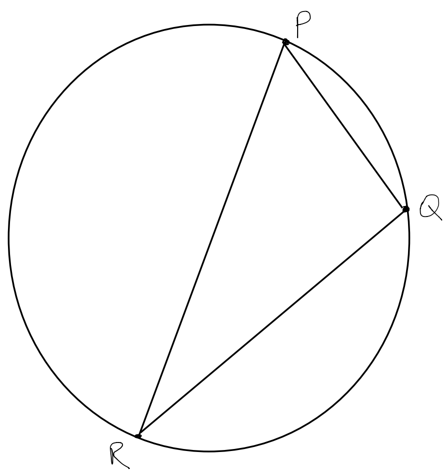


a) $m_{PQ}=\dfrac{10-1}{3-0}=\dfrac{9}{3}=3$

$m_{QR}=\dfrac{9-10}{6-3}=-\dfrac{1}{3}$

$m_{PQ}\cdot m_{QR}=-1$

$\therefore P\hat{Q}R$ is a right-angle AS REQUIRED

b) PR is a diameter

$\Rightarrow$ Circle centre is the midpoint of PR

$\left(\dfrac{0+6}{2},\dfrac{1+9}{2}\right)=(3,5)$

Radius $=\sqrt{(3-0)^2+(5-1)^2}=5$

$\therefore (x-3)^2+(y-5)^2=25$

$x^2-6x+9+y^2-10y+25=25$

$x^2+y^2-6x-10y+9=0$ AS REQUIRED

3. a) $x^2+y^2=64$ Centre $(0,0)$ Radius $=8$

   Distance $=\sqrt{(9-0)^2+(0-0)^2}=9>8 \therefore$ Outside

   b) $x^2+y^2-2x-6y-26=0$

   $x^2-2x+y^2-6y-26=0$

   $(x-1)^2-1+(y-3)^2-9-26=0$

   $(x-1)^2+(y-3)^2=36 \Rightarrow$ Centre $(1,3)$ Radius $=6$

   Distance $=\sqrt{(4-1)^2+(7-3)^2}=5<6 \therefore$ Inside

   c) $x^2+y^2+10x-4y=140$

   $x^2+10x+y^2-4y=140$

   $(x+5)^2-25+(y-2)^2-4=140$

   $(x+5)^2+(y-2)^2=169 \Rightarrow$ Centre $(-5,2)$ Radius $=13$

   Distance $=\sqrt{[7-(-5)]^2+(-3-2)^2}=13=13 \therefore$ Lies on the circle

   d) $x^2+y^2+2x+8y-13=0$

   $x^2+2x+y^2+8y-13=0$

   $(x+1)^2-1+(y+4)^2-16-13=0$

   $(x+1)^2+(y+4)^2=30 \Rightarrow$ Centre $(-1,-4)$ Radius $=\sqrt{30}$

   Distance $=\sqrt{[-4-(-1)]^2+[1-(-4)]^2}=\sqrt{34}>\sqrt{30} \therefore$ Outside

4. a) $x^2+y^2-6x+4y-12=0$

$x^2-6x+y^2+4y-12=0$

$(x-3)^2-9+(y+2)^2-4-12=0$

$(x-3)^2+(y+2)^2=25$

$\therefore A(3,-2)$

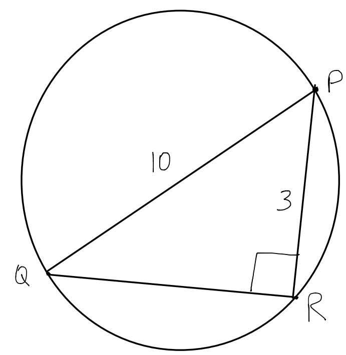
b) Radius $=\sqrt{25}=5$ AS REQUIRED

c) Length of $PQ=10 \Rightarrow PQ$ is a diameter

Using Pythagoras' theorem

$10^2=3^2+(QR)^2$

$QR=\sqrt{91}=9.5$



5. a) $x^2+y^2-10x+6y-15=0$

$x^2-10x+y^2+6y-15=0$

$(x-5)^2-25+(y+3)^2-9-15=0$

$(x-5)^2+(y+3)^2=49$

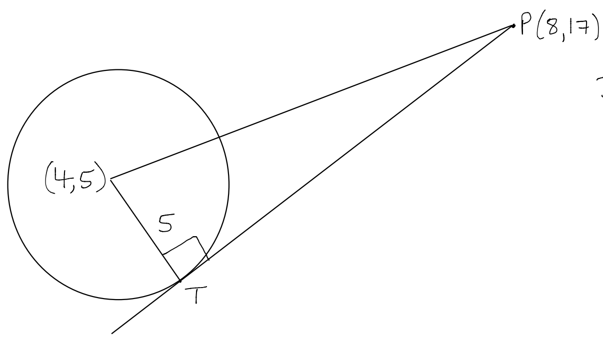
$\therefore$ Centre $(5,-3)$

b) Radius $=\sqrt{49}=7$

6. a) $a=4$ $b=5$ (You can see this if you draw the radius to the point where the circle touches the x-axis)

b) $(x-4)^2+(y-5)^2=25$

c)



Distance from Centre to P

$=\sqrt{(8-4)^2+(17-5)^2}=\sqrt{160}$

Using Pythagoras' theorem

$(\sqrt{160})^2=5^2+(PT)^2$

$PT=\sqrt{135}=11.6$

7. a) $(x-3)^2 + (y-4)^2 = (3\sqrt{2})^2$

b) Substitute $y = x+3$ $\Rightarrow$ $(x-3)^2 + (x+3-4)^2 = 18$

$$(x-3)^2 + (x-1)^2 = 18$$
$$x^2 - 6x + 9 + x^2 - 2x + 1 = 18$$
$$2x^2 - 8x - 8 = 0$$
$$x^2 - 4x - 4 = 0$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-4)}}{2}$$

$x = 2 + 2\sqrt{2}$ OR $x = 2 - 2\sqrt{2}$

$y = 5 + 2\sqrt{2}$ $\quad$ $y = 5 - 2\sqrt{2}$

$\therefore$ $(2+2\sqrt{2},\ 5+2\sqrt{2})$ $\quad$ $(2-2\sqrt{2},\ 5-2\sqrt{2})$

c) Distance $= \sqrt{[2+2\sqrt{2} - (2-2\sqrt{2})]^2 + [5+2\sqrt{2} - (5-2\sqrt{2})]^2} = 8$

8. $x^2 + y^2 + 6x + 2y = 27$

Substitute $y = 1-x$ $\Rightarrow$ $x^2 + (1-x)^2 + 6x + 2(1-x) = 27$

$$x^2 + 1 - 2x + x^2 + 6x + 2 - 2x = 27$$
$$2x^2 + 2x - 24 = 0$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$

$x = -4$ OR $x = 3$

$y = 5$ $\quad$ $y = -2$

$\therefore$ $A(-4, 5)$ $\quad$ $B(3, -2)$

$AB = \sqrt{[3-(-4)]^2 + (-2-5)^2} = \sqrt{98} = 7\sqrt{2}$

9. $x^2 + y^2 - 8x - 8y + 27 = 0$

Substitute $y = 2x+1$ $\Rightarrow$ $x^2 + (2x+1)^2 - 8x - 8(2x+1) + 27 = 0$

$$x^2 + 4x^2 + 4x + 1 - 8x - 16x - 8 + 27 = 0$$
$$5x^2 - 20x + 20 = 0$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)^2 = 0$$
$$x = 2$$

Repeated root, therefore $y = 2x+1$ is a tangent to the circle

Point of contact $(2, 5)$

10. $x^2+y^2-8x-16y+72=0$

Substitute $y=mx \Rightarrow x^2+(mx)^2-8x-16(mx)+72=0$

$$x^2+m^2x^2-8x-16mx+72=0$$

HARD!

$$(1+m^2)x^2+(-8-16m)x+72=0$$

Tangent $\Rightarrow$ Repeated root $\Rightarrow b^2-4ac=0$

$$\Rightarrow (-8-16m)^2-4(1+m^2)(72)=0$$
$$64+256m+256m^2-288-288m^2=0$$
$$-32m^2+256m-224=0$$
$$32m^2-256m+224=0$$
$$m^2-8m+7=0$$
$$(m-1)(m-7)=0$$
$$m=1 \quad \underline{\text{OR}} \quad m=7$$

11. a) Centre is the midpoint of AB $\Rightarrow \left(\frac{-5+3}{2}, \frac{6+8}{2}\right)=(-1,7)$

b) Radius $=\sqrt{(8-7)^2+[3-(-1)]^2}=\sqrt{17} \Rightarrow (x+1)^2+(y-7)^2=17$

c) $m_{\text{RADIUS AT A}}=\frac{6-7}{-5-(-1)}=\frac{-1}{-4}=\frac{1}{4} \Rightarrow m_{\text{TANGENT}}=-4$

$$\therefore y-6=-4(x+5)$$

12. a) $\left(\frac{-4+(-2)}{2}, \frac{9+(-5)}{2}\right)=(-3,2)$

b) Radius $=\sqrt{(9-2)^2+[-4-(-3)]^2}=\sqrt{50} \Rightarrow (x+3)^2+(y-2)^2=50$

c) Substitute $x=2, y=7 \Rightarrow (2+3)^2+(7-2)^2=5^2+5^2$

$=25+25=50 \quad \therefore R(2,7)$ lies on C AS REQUIRED

d) $P\hat{R}Q=90^\circ$ Angle in a semicircle

13. a) $m_{PR}=\frac{2-(-10)}{-10-(-2)}=\frac{12}{-8}=-3/2 \qquad m_{PQ}=\frac{14-2}{8-(-10)}=\frac{12}{18}=2/3$

$m_{PR}\cdot m_{PQ}=-1 \Rightarrow$ PR is perpendicular to PQ AS REQUIRED

b) RQ is a diameter $\Rightarrow$ Centre $\left(\frac{8+(-2)}{2}, \frac{14+(-10)}{2}\right)=(3,2)$

Radius $=\sqrt{(14-2)^2+(8-3)^2}=13$

$$\therefore (x-3)^2+(y-2)^2=13^2$$
$$x^2-6x+9+y^2-4y+4=169$$
$$x^2+y^2-6x-4y-156=0 \quad \text{AS REQUIRED}$$