C2 - Chapter 5 The binomial expansion - Extra practice - Solutions

1. $(1-2x)^4 = 1+4(-2x) + \frac{4.3}{2!}(-2x)^2 + \frac{4.3.2}{3!}(-2x)^3 + \frac{4.3.2.1}{4!}(-2x)^4$

$= 1-8x+24x^2-32x^3+16x^4$

2. $(2+3x)^9 = 2^9(1+3x/2)^9$

$= 512\left\{1+9\left(\frac{3x}{2}\right)+\frac{9.8}{2!}\left(\frac{3x}{2}\right)^2+\frac{9.8.7}{3!}\left(\frac{3x}{2}\right)^3+\dots\right\}$

$= 512\left(1+\frac{27x}{2}+81x^2+\frac{567}{2}x^3+\dots\right)$

$= 512+6912x+41472x^2+145152x^3+\dots$

3. $(2+x)(1-x)^5 = (2+x)\left\{1+5(-x)+\frac{5.4}{2!}(-x)^2+\frac{5.4.3}{3!}(-x)^3+\dots\right\}$

$= (2+x)(1-5x+10x^2-10x^3+\dots)$

$= 2-10x+20x^2-20x^3+x-5x^2+10x^3+\dots$

$= 2-9x+15x^2-10x^3+\dots$

4. $(2+px)^7 = 2^7(1+px/2)^7$

$= 128\left\{1+7(px/2)+\frac{7.6}{2!}(px/2)^2+\dots\right\}$

$= 128\left(1+\frac{7px}{2}+\frac{21}{4}p^2x^2+\dots\right)$

$= 128+448px+672p^2x^2+\dots$

$672p^2 = 6048$

$p^2 = 9 \Rightarrow p = \pm 3$

5. $(1+px)^5 = 1+5px+\frac{5.4}{2!}(px)^2+\frac{5.4.3}{3!}(px)^3+\dots$

$= 1+5px+10p^2x^2+10p^3x^3+\dots$

$10p^3 = 2\cdot 10p^2$

$10p^3-20p^2=0 \Rightarrow 10p^2(p-2)=0 \Rightarrow p=0$ OR $p=2$

6. $(1+px)^{12} = 1 + 12px + \frac{12 \cdot 11}{2!}(px)^2 + \dots$

$= 1 + 12p + 66p^2x^2 + \dots$

Now, $12p = 2q$ and $66p^2 = 55q$

$\Rightarrow 6p = q$ $\quad 6p^2 = 5q$

$6p^2 = 5 \cdot 6p$

$6p^2 - 30p = 0$

$6p(p-5) = 0 \Rightarrow p = 0$ OR $p = 5$

Reject $\quad q = 30$

7. $\binom{a}{5} = \frac{7!}{5!\,2!} = c \quad a = 7 \quad b = 2 \quad c = 21$ Results follow straight away from the definition of ${}^nC_r$.

8. a) $(2-3x)^7 = 2^7(1 - 3x/2)^7$

$= 128\left\{1 + 7(-3x/2) + \frac{7 \cdot 6}{2!}(-3x/2)^2 + \frac{7 \cdot 6 \cdot 5}{3!}\left(-\frac{3x}{2}\right)^3 + \dots\right\}$

$= 128\left(1 - 21x/2 + 189x^2/4 - 945x^3/8 + \dots\right)$

$= 128 - 1344x + 6048x^2 - 15120x^3 + \dots$

b) $2 - 3x = 1.94 \Rightarrow x = 0.02$

$\Rightarrow 1.94^7 \simeq 128 - 1344(0.02) + 6048(0.02)^2 - 15120(0.02)^3$

$= 103.41824$

9a) $(1+ax)^n = 1 + nax + \frac{n(n-1)}{2!}(ax)^2 + \frac{n(n-1)(n-2)}{3!}(ax)^3 + \dots$

b) $na = 8 \quad \frac{n(n-1)a^2}{2} = 30$

$a = 8/n$ Substitute $a = 8/n \Rightarrow \frac{n(n-1)(8/n)^2}{2} = 30$

$n(n-1)\frac{64}{n^2} = 60$

$\frac{64(n-1)}{n} = 60 \Rightarrow 64n - 64 = 60n$

$\Rightarrow n = 16 \Rightarrow a = 1/2$

c) Coefficient of $x^3 = \frac{n(n-1)(n-2)}{3!}a^3 = \frac{16 \cdot 15 \cdot 14}{3!}\left(\frac{1}{2}\right)^3 = 70$

10. $(1+px)^n = 1 + npx + \frac{n(n-1)}{2!}(px)^2 + \ldots$

$np = 18 \qquad \frac{n(n-1)}{2}\cancel{p^2} = 36\cancel{p^2}$

$n(n-1) = 72$

$n^2 - n - 72 = 0$

$(n-9)(n+8) = 0 \quad \Rightarrow n = 9 \quad \underline{\underline{OR}} \quad n = -8$

$\Rightarrow p = 2$ Reject since n is positive

11. a) $(1+3x)^n = 1 + 3nx + \frac{n(n-1)}{2!}(3x)^2 + \frac{n(n-1)(n-2)}{3!}(3x)^3 + \frac{n(n-1)(n-2)(n-3)}{4!}(3x)^4 + \ldots$

b) $\frac{\cancel{n(n-1)}(n-2)}{3!}3^{\cancel{3}} = 10\frac{\cancel{n(n-1)}}{2!}\cancel{3^2}$

$\frac{\cancel{3}^{1}(n-2)}{\cancel{6}_{2}} = \frac{10}{2} \quad \Rightarrow \quad n-2 = 10 \Rightarrow n = 12$

c) Coefficient of x^4: $\frac{12 \cdot 11 \cdot 10 \cdot 9}{4!}3^4 = 40095$

12. $(2-px)^6 = 2^6\left(1 - \frac{px}{2}\right)^6$

$= 64\left\{1 + 6\left(-\frac{px}{2}\right) + \frac{6 \cdot 5}{2!}\left(-\frac{px}{2}\right)^2 + \ldots\right\}$

$= 64\left\{1 - 3px + \frac{15p^2}{4}x^2 + \ldots\right\}$

$= 64 - 192px + 240p^2x^2 + \ldots$

$240p^2 = 135$

$p^2 = \frac{9}{16} \quad \Rightarrow p = \frac{3}{4} \quad \underline{\underline{OR}} \quad p = -\frac{3}{4}$

Reject since $p > 0$

$A = -192p = -192\left(\frac{3}{4}\right) = -144$