C2 - Chapter 5 - The binomial expansion

* $n! = n(n-1)(n-2)(n-3)\ldots 4\cdot 3\cdot 2\cdot 1$

* $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$ This corresponds to the number of ways of choosing r objects from a collection of n objects

eg From a group of 7 people how many committees of 4 persons can we form? ${}^7C_4 = 35$ different committees.

* $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$

A few points to note: ① The bracket should be in the form

Make sure this is 1 $(1 \pm \square)^n$ something in x, y, z etc

② The factorial term and the power of x agree. eg $\frac{n(n-1)(n-2)}{3!}x^3$

Examples (in each case the first three terms are given)

i) $(1+2x)^4 = 1 + 4(2x) + \frac{4\cdot 3}{2!}(2x)^2 + \ldots = 1 + 8x + 24x^2 + \ldots$

ii) $(1 - x/3)^5 = 1 + 5(-x/3) + \frac{5\cdot 4}{2!}(-x/3)^2 + \ldots = 1 - \frac{5x}{3} + \frac{10x^2}{9} + \ldots$

iii) $(2+x)^6 = [2(1+x/2)]^6 = 2^6(1+x/2)^6 = 64\left\{1 + 6(x/2) + \frac{6\cdot 5}{2!}(x/2)^2 + \ldots\right\}$

$= 64\left\{1 + 3x + \frac{15x^2}{4} + \ldots\right\}$

$= 64 + 192x + 230x^2 + \ldots$

iv) $(1-x)(3+2x)^3 = (1-x)\,3^3\left(1+\frac{2x}{3}\right)^3$

$= 27(1-x)\left\{1 + 3\left(\frac{2x}{3}\right) + \frac{3\cdot 2}{2!}\left(\frac{2x}{3}\right)^2 + \ldots\right\}$

$= 27(1-x)\left(1 + 2x + \frac{4x^2}{3} + \ldots\right) = (27 - 27x)\left(1 + 2x + \frac{4x^2}{3} + \right.$

$= 27 + 54x + 36x^2 - 27x - 54x^2 = 27 + 27x - 18x^2 + \ldots$

v) $(4+x)^6(1-4x)^4 = 4^6(1+x/4)^6(1-4x)^4$

HARD!

$= 4096\left\{1 + 6(x/4) + \frac{6\cdot5}{2!}(x/4)^2 + \ldots\right\}\left\{1 + 4(-4x) + \frac{4\cdot3}{2!}(-4x)^2 + \ldots\right\}$

$= 4096\left(1 + 3x/2 + 15x^2/16 + \ldots\right)\left(1 - 16x + 96x^2 + \ldots\right)$

$= 4096\left(1 - 16x + 96x^2 + 3x/2 - 24x^2 + 15x^2/16 + \ldots\right)$

$= 4096\left(1 - 29x/2 + 87x^2 + \ldots\right)$

$= 4096 - 59392x + 356352x^2 + \ldots$

Only consider the pairs that will give you terms that you will keep

* % error $= \dfrac{|\text{Exact} - \text{Estimate}|}{\text{Exact}} \times 100\%$

Remember that this cannot be negative.

* Agreement to some level of accuracy

Compare the two numbers and decide to which decimal place or significant number, when rounded will give the same number

Examples

i) 3.124893768 3.12539 Agreement to 3dp's or 4sf since both numbers round to 3.125

ii) 0.032 0.02948 Agreement to 2dp's or 1sf since both numbers round to 0.03

iii) 11.8984837 11.8976344 Agreement to 3dp's or 5sf since both numbers round to 11.898