C2 - Chapter 6 - Radian measure and its applications - Extra practice - Solutions

1\. a) M has coordinates $\left(\frac{0+28}{2}, \frac{0+0}{2}\right) = (14,0)$

$BM = \sqrt{(7-14)^2 + (24-0)^2} = \sqrt{(-7)^2 + 24^2} = \sqrt{625} = 25$ mm AS REQUIRED

b) $AB = \sqrt{(7-0)^2 + (24-0)^2} = 25$

Since $AB = BM$, ΔABM is isosceles

$\Rightarrow \cos(B\hat{M}A) = 7/25$ (Drop the perpendicular from B to AM to create a right-angled triangle)

$\Rightarrow B\hat{M}A = 1.287002218$

Similarly, $C\hat{M}D = B\hat{M}A$

$\Rightarrow B\hat{M}C = \pi - 2 \cdot 1.287002218 = 0.5675882184$

$= 0.568$ (to 3sf) AS REQUIRED

c) Area of cross-section = Area of ΔABM + Area of sector MBC + Area of ΔMCD

$= \frac{24 \times 14}{2} + \frac{1}{2} 25^2 \cdot 0.5675882184 + \frac{24 \times 14}{2}$

$= 513.3713183 = 513$ mm^2 (to 3sf)

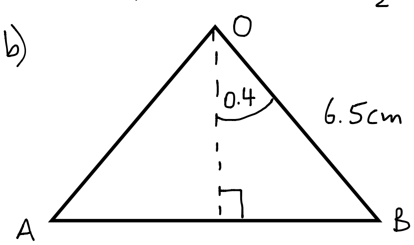
d) Volume = Area of cross-section × length

$= 513.3713183 \times 85 = 43636.56205$ mm^3 $= 43.63656205$ cm^3

$= 44$ cm^3 (to the nearest cm^3)

2\. a) Area of sector $AOB = \frac{1}{2} \cdot 6.5^2 \cdot 0.8 = 16.9$ cm^2

b)



$AB = 2 \cdot 6.5 \sin(0.4) = 5.06243845$

$= 5.06$ (to 3sf) AS REQUIRED

c) Perimeter = Length of chord AB + length of arc AB $= 5.06243845 + 6.5 \cdot 0.8$

$= 10.3$ cm (to 3sf)

3. a) $15 = \frac{1}{2} r^2 \, 1.5 \Rightarrow r^2 = 20 \Rightarrow r = \sqrt{20} = 2\sqrt{5}$ AS REQUIRED

b) Perimeter of sector OAB $= 2\sqrt{5} + 2\sqrt{5} \cdot 1.5 + 2\sqrt{5} = 7\sqrt{5} = 15.7\text{cm}$ (to 3sf)

c) $R = \frac{1}{2}(2\sqrt{5})^2 (1.5 - \sin 1.5) = 5.025050134 = 5.025$ (to 3dp's)

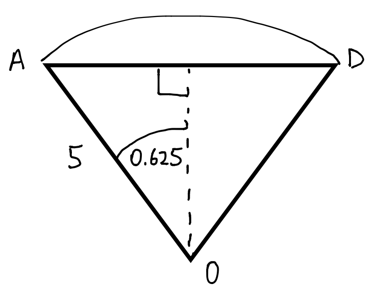
4. a) Area of flowerbed $= \frac{1}{2}7^2\theta - \frac{1}{2}5^2\theta = 12\theta$

b) $15 = 12\theta \Rightarrow \theta = \frac{15}{12} = 1.25$ AS REQUIRED

c) Perimeter of flowerbed = AB + arc BC + CD + arc DA

$= 2 + 7 \cdot 1.25 + 2 + 5 \cdot 1.25 = 19 \text{ m}$

d)



$AD = 2 \cdot 5 \sin 0.625$

$= 5.850972729$

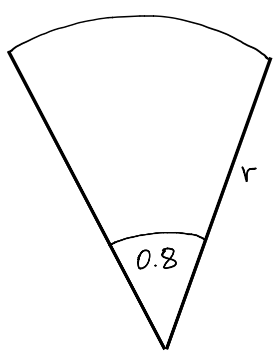
Length of arc $AD = 5 \cdot 1.25 = 6.25$

$\therefore$ Reduction in perimeter $= 6.25 - 5.850972729$

$= 0.3990272706 \text{ m}$

$= 40\text{cm}$ (to the nearest cm)

5. a)



$P = r + r + 0.8r = 2.8r$

$A = \frac{1}{2} r^2 \cdot 0.8 = 0.4r^2$

$A + P = 31.2 \Rightarrow 0.4r^2 + 2.8r = 31.2$

$r^2 + 7r - 78 = 0$ AS REQUIRED

b) $(r+13)(r-6) = 0$

$r = -13$ OR $r = 6$

Reject, not valid $\Rightarrow P = 2.8 \cdot 6 = 16.8\text{cm}$ $\quad A = 31.2 - 16.8 = 14.4 \text{ cm}^2$

6. a) Perimeter of semi-circle AB $= x + \frac{2\pi(x/2)}{2} = x + \frac{\pi x}{2}$

Perimeter of sector OCD $= x + x + x\theta = 2x + x\theta$

$$2\cancel{x} + \cancel{x}\theta = \cancel{x} + \frac{\pi \cancel{x}}{2}$$

$$2 + \theta = 1 + \pi/2$$

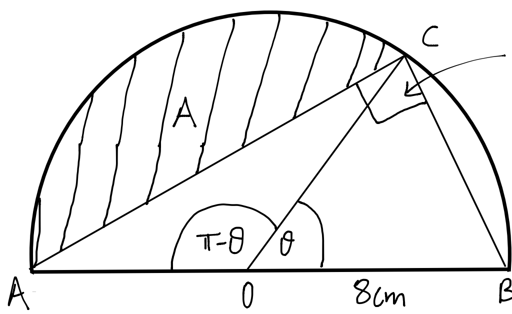
$$\theta = \pi/2 - 1 \quad \text{AS REQUIRED}$$

b) If $x = 6$, Area of semi-circle AB $= \frac{\pi \cdot 3^2}{2} = \frac{9\pi}{2}$

Area of sector OAB $= \frac{1}{2} 6^2 \left(\frac{\pi}{2} - 1\right) = 9\pi - 18$

$\Rightarrow$ Difference in areas $= \frac{9\pi}{2} - (9\pi - 18) = 3.86$ cm (to 3sf)

7.



angle in a semicircle is always 90°

$A = \frac{1}{2} 8^2 \left[\pi - \theta - \sin(\pi - \theta)\right]$

$= 32 (\pi - \theta - \sin\theta)$ $\sin(\pi - \theta) = \sin\theta$ You will learn this later in C2

$B = \frac{1}{2} 8^2 \sin\theta = 32 \sin\theta$

$$\therefore \quad 32(\pi - \theta - \sin\theta) = 2 \cdot 32 \sin\theta$$

$$32\pi - 32\theta - 32\sin\theta = 64\sin\theta$$

$$32\pi = 32\theta + 96\sin\theta$$

$$\pi = \theta + 3\sin\theta \quad \text{AS REQUIRED}$$

8. a) Area of shaded region = Area of sector − Area of Δ OCD

$= \frac{1}{2} 10^2 \cdot 0.75 - \frac{1}{2} 5 \cdot 4 \cdot \sin 0.75 = 30.7 \text{ cm}^2$ (to 3sf)

b) $(CD)^2 = 5^2 + 4^2 - 2 \cdot 5 \cdot 4 \cos 0.75 \Rightarrow CD = 3.425265719$

Perimeter = CA + arc AB + BD + DC $= 5 + 10 \cdot 0.75 + 6 + 3.425265719$

$= 21.9$ cm (to 3sf)