C2 - Chapter 7 - Geometric sequences and series - Extra practice - Solutions

1. $U_1 = 5 = a \qquad r = 1.2$

a) $U_{16} = 5 \times 1.2^{15} = 77$ (to the nearest integer)

b) $S_{30} = \frac{5(1-1.2^{30})}{1-1.2} = 5909$ (to the nearest integer)

c) $r = 1.2 > 1$ $\therefore$ It does not converge and has no sum to infinity

2 a) $U_2 = 9 = ar \qquad U_5 = 1.125 = ar^4$

$$\frac{U_5}{U_2} = \frac{ar^4}{ar} = \frac{1.125}{9} \Rightarrow r^3 = \frac{1}{8} \Rightarrow r = \frac{1}{2}$$

b) $9 = a \cdot \frac{1}{2} \Rightarrow a = 18$

c) $S_\infty = \frac{18}{1-\frac{1}{2}} = 36$

3. a) $U_1 = a = 1200 \qquad S_\infty = 960$

$$960 = \frac{1200}{1-r} \Rightarrow 1-r = \frac{1200}{960} \Rightarrow 1-r = 1.25 \Rightarrow r = -\frac{1}{4} \text{ AS REQUIRED}$$

b) $U_9 - U_{10} = ar^8 - ar^9 = 1200 \cdot (-\frac{1}{4})^8 - 1200 \cdot (-\frac{1}{4})^9 = 0.023$ (to 3dp's)

c) $S_n = \frac{1200\left(1-(-\frac{1}{4})^n\right)}{1-(-\frac{1}{4})}$

d) $S_n = 960\left[1-(-\frac{1}{4})^n\right]$ if n is odd the $(-\frac{1}{4})^n = -(\frac{1}{4})^n$

Hence, $S_n = 960\left[1--(\frac{1}{4})^n\right] = 960\left[1+(\frac{1}{4})^n\right]$ AS REQUIRED

4. a) $U_{n+1} = \frac{1}{3}U_n \qquad U_1 = 1 \qquad U_2 = \frac{1}{3} \cdot 1 = \frac{1}{3}$

$U_3 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \qquad U_4 = \frac{1}{3} \cdot \frac{1}{9} = \frac{1}{27}$

b) $\sum_{r=1}^{\infty} U_r = \underbrace{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots}_{\text{sum to infinity of a geometric progression with } a=1 \text{ and } r=\frac{1}{3}} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

5. a) $U_1 + U_2 = 10.8$ $\quad U_3 + U_4 = 43.2$

$= a + ar$ $\quad = ar^2 + ar^3$

$= a(1+r)$ $\quad = ar^2(1+r)$

$43.2 = ar^2(1+r)$

$10.8 = a(1+r)$

$\Rightarrow \frac{43.2}{10.8} = r^2 \quad \Rightarrow r = \sqrt{\frac{43.2}{10.8}} = 2$ (since r must be positive for all terms to be positive)

$10.8 = a(1+2) \Rightarrow a = 3.6$

b) $S_{16} = \frac{3.6(1-2^{16})}{1-2} = 235926$

6. $\sum_{r=3}^{10} 3^r = \underbrace{27 + 81 + 243 + \ldots}_{\text{Sum of first 8 (10-3+1) terms of a GP with } a=27 \text{ and } r=3} = \frac{27(1-3^8)}{1-3} = 88560$

7. a) $U_1 = K+10 \quad U_2 = K \quad U_3 = K-6$

$\frac{U_3}{U_2} = \frac{U_2}{U_1} \Rightarrow \frac{K-6}{K} = \frac{K}{K+10}$

$(K-6)(K+10) = K^2$

$K^2 + 4K - 60 = K^2$

$4K = 60 \Rightarrow K = 15$

b) $U_1 = 15+10 = 25 \quad r = \frac{K}{K+10} = \frac{15}{15+10} = \frac{3}{5}$

$S_\infty = \frac{25}{1-3/5} = 62.5$

8. a) $U_1 = 4 \quad U_2 = 4 \times 1.25 = 5 \quad U_3 = 5 \times 1.25 = 6.25 \Rightarrow a = 4 \quad r = 1.25$

$U_8 = 4 \times 1.25^7 = 19.1$ mm (to 3sf)

b) $S_{20} = \frac{4(1-1.25^{20})}{1-1.25} = 1371.778781$ mm $= 1.37$ m (to 3sf)

9a) $U_1 = 64 = a$ $\quad U_3 - U_2 = 20$

$= ar^2 - ar$

$= ar(r-1)$

$20 = 64r(r-1)$

$5 = 16r(r-1)$

$5 = 16r^2 - 16r$

$16r^2 - 16r - 5 = 0$ AS REQUIRED

b) $16r^2 - 16r - 5 = 0$

$(4r+1)(4r-5) = 0 \Rightarrow r = -1/4$ OR $r = 5/4$

c) For $r = -1/4$, $U_4 = 64 \cdot (-1/4)^3 = -1$

For $r = 5/4$, $U_4 = 64 \cdot (5/4)^3 = 125$

d) Series converges for $r = -1/4 \Rightarrow S_\infty = \frac{64}{1-(-1/4)} = 51.2$

10. a) $r = 1.5$ $\quad U_3 = 18 = ar^2$

$18 = a \cdot 1.5^2 \Rightarrow a = 8 = U_1$

b) $S_6 = \frac{8(1-1.5^6)}{1-1.5} = 166.25$

c) $8 \cdot 1.5^{K-1} > 8000$

$1.5^{K-1} > 1000$

$(K-1)\log 1.5 > \log 1000$

$K-1 > \frac{\log 1000}{\log 1.5}$

$K > \frac{\log 1000}{\log 1.5} + 1 \Rightarrow K > 18.0366... \Rightarrow K = 19$

11.a) Number of new subscribers in

1st week	2nd week	3rd week	...	nth week
200	200×1.15	200×1.15^2	...	$200 \times 1.15^{n-1}$

$\Rightarrow$ In fourth week $200 \times 1.15^3 = 304$ new subscribers AS REQUIRED

b) Total number of subcribers after 10 weeks $= 3600 + S_{10} = 3600 + \frac{200(1-1.15^{10})}{1-1.15}$

$= 7661$ subscribers