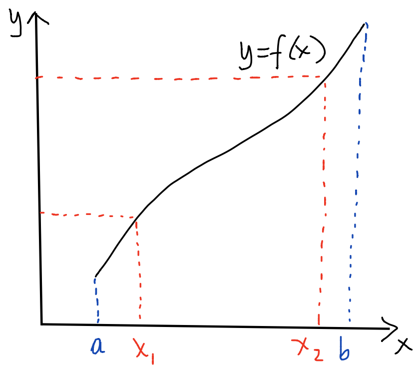
C2 - Chapter 9 - Differentiation - Summary

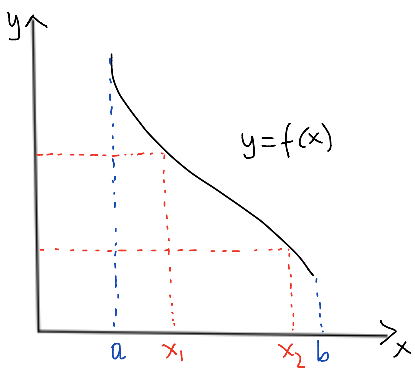
* A function $f(x)$ is said to be increasing in the interval (a,b) if for any two values in the interval, say x_1 and x_2, such that $x_1 < x_2$, $f(x_1) < f(x_2)$



In simple words this means that when you draw the graph of $f(x)$ in the interval (a,b) you only "go upwards".

It follows, that $f'(x) > 0$ for $a \leq x \leq b$.

* A function $f(x)$ is said to be decreasing in the interval (a,b) if for any two values in the interval, say x_1 and x_2, such that $x_1 < x_2$, $f(x_1) > f(x_2)$



In simple words this means that when you draw the graph of $f(x)$ in the interval (a,b) you only "go downwards".

It follows, that $f'(x) < 0$ for $a \leq x \leq b$.

* Points of zero gradient (ie $\frac{dy}{dx} = 0$) are called stationary or turning points and they can be minimum, maximum or points of inflexion.

* To determine the nature of a stationary point determine and evaluate $\frac{d^2y}{dx^2}$.
  - If $\frac{d^2y}{dx^2} > 0$ then it is a minimum point
  - If $\frac{d^2y}{dx^2} < 0$ then it is a maximum point
  - If $\frac{d^2y}{dx^2} = 0$ then determine and evaluate $\frac{d^3y}{dx^3}$.
    - If $\frac{d^3y}{dx^3} \neq 0$ then it is a point of inflexion
    - If $\frac{d^3y}{dx^3} = 0$ then you need to draw up a table and investigate the gradient of the curve to the left and right of the stationary point

* Table method.

e.g. Find all stationary points on the curve $y = 2x^3 - 9x^2 + 12x$ and determine their nature

$\frac{dy}{dx} = 6x^2 - 18x + 12$ $\qquad$ $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 18x + 12 = 0$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$x = 1$ OR $x = 2$

x	0.9	1	1.1
$\frac{dy}{dx}$	0.11	0	-0.09

x	1.9	2	2.1
$\frac{dy}{dx}$	-0.09	0	0.11

$\therefore$ At $x=1$ we have a maximum point

At $x=2$ we have a minimum point

* NOTE: When evaluating $\frac{d^2y}{dx^2}$ to classify a turning point make sure you say explicitly $\frac{d^2y}{dx^2} > 0$ or $\frac{d^2y}{dx^2} < 0$ to justify your conclusion.