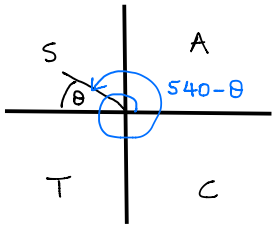
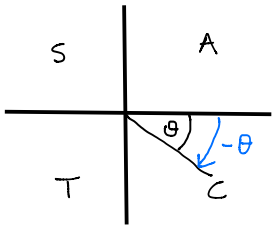
C2 - Chapters 8 and 10 - Trigonometry - Extra practice - Solutions

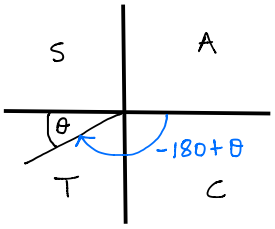
1. a) $\sin(540-\theta) = \sin\theta$



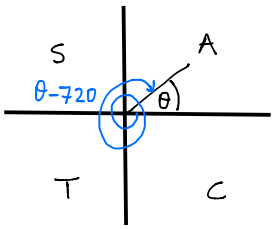
b) $\cos(-\theta) = \cos\theta$



c) $\tan(-180+\theta) = \tan\theta$



d)



2 a) $\tan x = 5$

$\alpha = \tan^{-1}(5) = 78.6900675^3$

$x = 180n + 78.6900675^3$

$x = 78.7, 258.7$

b) $3\cos^2 y + 8\sin y = 0$

$3(1-\sin^2 y) + 8\sin y = 0$

$3 - 3\sin^2 y + 8\sin y = 0$

$3\sin^2 y - 8\sin y - 3 = 0$

Let $w = \sin y \Rightarrow$ $3w^2 - 8w - 3 = 0$

$(3w+1)(w-3) = 0$

$w = -1/3$ OR $w = 3$

$\sin y = -1/3$ $\sin y = 3$

$\alpha = \sin^{-1}(-1/3)$ Reject, not valid

$\alpha = -0.339837$

$y = 2\pi n - 0.339837$

$y = 2\pi n + \pi + 0.339837$

$y = 3.48, 5.94$

3. $\sin(\theta - 30) = 0.7$

$\alpha = \sin^{-1}(0.7) = 44.424383$

$\theta - 30 = 360n + 44.424383 \Rightarrow \theta = 360n + 74.424383$

$\theta - 30 = 360n + 180 - 44.424383 \Rightarrow \theta = 360n + 105.575617$

$\theta = 74.4, 105.6$

4. $LHS = (\cos x + \sin x)^2 + (\cos x - \sin x)^2$

$= (\cos x + \sin x)(\cos x + \sin x) + (\cos x - \sin x)(\cos x - \sin x)$

$= \cos^2 x + \cos x \sin x + \sin x \cos x + \sin^2 x + \cos^2 x - \cos x \sin x - \sin x \cos x + \sin^2 x$

$= \cos^2 x + \sin^2 x + \cos^2 x + \sin^2 x$

$= 1 + 1 = 2 = RHS$ AS REQUIRED

5. $\sin(2x + 50) = 0.6$

$\alpha = \sin^{-1}(0.6) = 36.869898$

$2x + 50 = 360n + 36.869898 \Rightarrow x = 180n - 6.565$

$2x + 50 = 360n + 180 - 36.869898 \Rightarrow x = 180n + 46.565$

$x = 46.6, 173.4$

6. a) $\sin 2x = 1/2$

$\alpha = \sin^{-1}(1/2) = 30$

$2x = 360n + 30 \Rightarrow x = 180n + 15$

$2x = 360n + 180 - 30 \Rightarrow x = 180n + 75$

$x = -165, -105, 15, 75$

b) $\sin^2(3x/2) = 1/2$

$\sin(3x/2) = \pm\sqrt{1/2}$

$\sin(3x/2) = 1/\sqrt{2}$ OR $\sin(3x/2) = -1/\sqrt{2}$

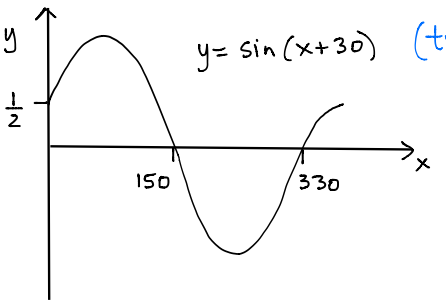
$\alpha = \sin^{-1}(1/\sqrt{2}) = 45$ $\alpha = \sin^{-1}(-1/\sqrt{2}) = -45$

$\frac{3x}{2} = 360n + 45 \Rightarrow x = 240n + 30$ $\frac{3x}{2} = 360n - 45 \Rightarrow x = 240n - 30$

$\frac{3x}{2} = 360n + 180 - 45 \Rightarrow x = 240n + 90$ $\frac{3x}{2} = 360n + 180 + 45 \Rightarrow x = 240n + 150$

$x = -150, -90, -30, 30, 90, 150$

7. a)



$y = \sin(x+30)$ (translation 30° to the left)

b) Cut on y-axis $\Rightarrow x=0 \Rightarrow y = \sin 30 = \frac{1}{2}$

c) $\sin(x+30) = -\frac{1}{2}$

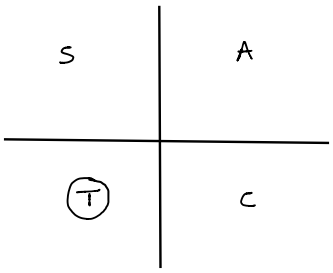
$\alpha = \sin^{-1}(-\frac{1}{2}) = -30$

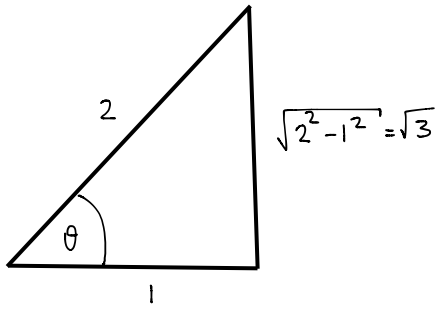
$x+30 = 360n - 30 \Rightarrow x = 360n - 60$

$x+30 = 360n + 180 + 30 \Rightarrow x = 360n + 180$

$x = 180, 300$

8.





$\sin\theta = -\sqrt{3}/2 \qquad \tan\theta = \sqrt{3}$

9 a) Cuts the y-axis $\Rightarrow x=0 \Rightarrow y = 2\sin(5\pi/6) = 1 \qquad \therefore (0,1)$

b) $2\sin(2x + 5\pi/6) = \sqrt{2}$

$\sin(2x + 5\pi/6) = \sqrt{2}/2$

$\alpha = \sin^{-1}(\sqrt{2}/2) = \pi/4$

$2x + 5\pi/6 = 2\pi n + \pi/4 \Rightarrow x = \pi n - 7\pi/12$

$2x + 5\pi/6 = 2\pi n + \pi - \pi/4 \Rightarrow x = \pi n - \pi/12$

$x = 5\pi/12, 11\pi/12, 17\pi/12, 23\pi/12$

10 a) LHS $= (3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$

$= 9\sin^2 x + 3\sin x\cos x + 3\cos x\sin x + \cos^2 x + \sin^2 x - 3\sin x\cos x - 3\cos x\sin x + 9\cos^2 x$

$= 10\sin^2 x + 10\cos^2 x$

$= 10(\sin^2 x + \cos^2 x) = 10 = \text{RHS}$ AS REQUIRED

b) LHS $= \frac{1}{\sin x} - \sin x$

$= \frac{1 - \sin^2 x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$

$= \cos x \cdot \frac{\cos x}{\sin x}$

$= \cos x \cdot \frac{1}{\tan x} = \frac{\cos x}{\tan x} = \text{RHS}$ AS REQUIRED

11. a) $\cos(\theta - 10) = \cos 15$

$\alpha = \cos^{-1}(\cos 15) = 15$

$\theta - 10 = 360n \pm 15$

$\theta - 10 = 360n + 15$ OR $\theta - 10 = 360n - 15$

$\theta = 360n + 25$ $\theta = 360n - 5$

$\theta = 25, 355$

b) $\tan 2\theta = 0.4$

$\alpha = \tan^{-1}(0.4) = 21.80140949$

$2\theta = 180n + 21.80140949$

$\theta = 90n + 10.90070474$

$\theta = 10.9, 100.9, 190.9, 280.9$

c) $2\sin\theta\tan\theta = 3$

$2\sin\theta \cdot \frac{\sin\theta}{\cos\theta} = 3$

$2\sin^2\theta = 3\cos\theta$

$2(1 - \cos^2\theta) = 3\cos\theta$

$2 - 2\cos^2\theta - 3\cos\theta = 0$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

Let $y = \cos\theta \Rightarrow 2y^2 + 3y - 2 = 0$

$(2y-1)(y+2)=0$

$y=\frac{1}{2}$ OR $y=-2$

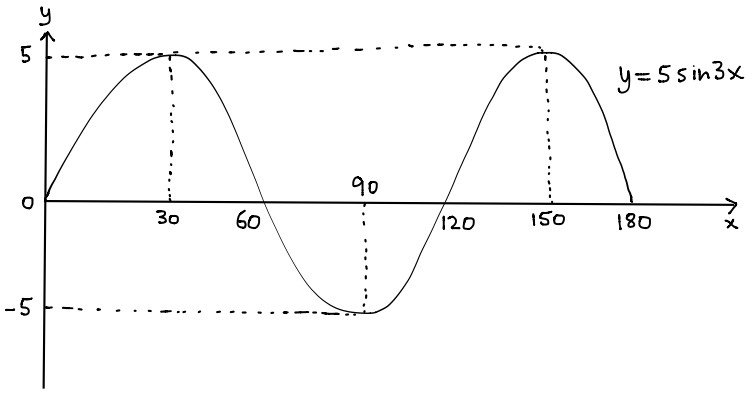
$\cos\theta=\frac{1}{2}$ $\cos\theta=-2$

$\alpha=\cos^{-1}(\frac{1}{2})=60$ Reject, not valid

$\theta=360n\pm 60$

$\theta=60,300$

12. a)



b) Maximum: (30,5) (150,5) Minimum (90,-5)

c) $5\sin3x=2.5$

$\sin3x=\frac{1}{2}$

$\alpha=\sin^{-1}(\frac{1}{2})=30$

$3x=360n+30 \Rightarrow x=120n+10$

$3x=360n+180-30 \Rightarrow x=120n+50$

$x=10,50,130,170,250,290$

13 a) $3\sin x=8\cos x$

$\frac{3\sin x}{\cos x}=8 \Rightarrow \tan x=\frac{8}{3}$

b) $3\sin x-8\cos x=0$

$3\sin x=8\cos x$

$\tan x=\frac{8}{3}$

$\alpha=\tan^{-1}(\frac{8}{3})=69.44395478$

$x=180n+69.44395478$

$x=69.4, 249.4$

c) $3\sin^2 y - 8\cos y = 0$

$3(1-\cos^2 y) - 8\cos y = 0$

$3 - 3\cos^2 y - 8\cos y = 0$

$3\cos^2 y + 8\cos y - 3 = 0$

Let $w = \cos y \Rightarrow 3w^2 + 8w - 3 = 0$

$(3w-1)(w+3) = 0$

$w = 1/3$ OR $w = -3$

$\cos y = 1/3$ $\cos y = -3$

$\alpha = \cos^{-1}(1/3) = 109.47122$ Reject, not valid

$y = 360n \pm 109.47122$

$y = 109.5, 250.5$

14. a) A (0,1) B (45,0) C (270,-1)

b) Stretch along the x-axis, scale factor 1/2.

c) $\cos 2x = 0.37$

$\alpha = \cos^{-1}(0.37) = 68.28438272$

$2x = 360n \pm 68.28438272$

$x = 180n \pm 34.14219136$

$x = 34.1, 145.9, 214.1, 325.9$

15 a) $\sin x = 0.8$

$\alpha = \sin^{-1}(0.8) = 0.927295218$

$x = 2\pi n + 0.927295218$

$x = 2\pi n + \pi - 0.927295218$

$x = 0.923, 2.21$

b) i) $M(3\pi/2, -1)$

ii) $\pi - \alpha$

iii) Length of RS = length of PQ = $\pi - \alpha - \alpha = \pi - 2\alpha$

c)