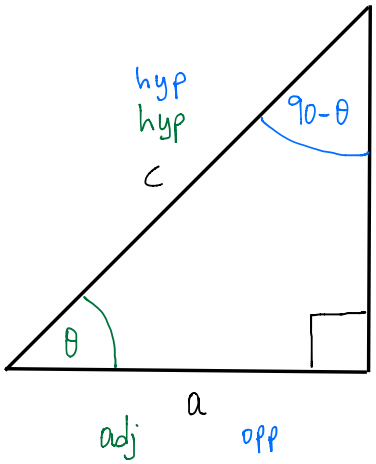
C2 - Chapters 8 and 10 - Trigonometry - Summary



Reminder

① $\sin\theta = \frac{opp}{hyp}$ $\quad \cos\theta = \frac{adj}{hyp}$ $\quad \tan\theta = \frac{opp}{adj}$

SOH CAH TOA

Some Old Horses Can Always Hide Their Old Ages

② Pythagoras' theorem

$a^2 + b^2 = c^2$

* $\sin\theta = \frac{b}{c}$ $\quad \sin(90-\theta) = \frac{a}{c}$

$\cos\theta = \frac{a}{c}$ $\quad \cos(90-\theta) = \frac{b}{c}$

$\tan\theta = \frac{b}{a}$ $\quad \tan(90-\theta) = \frac{a}{b} \Rightarrow \frac{1}{\tan(90-\theta)} = \frac{b}{a}$

$\sin\theta = \cos(90-\theta)$ $\quad$ eg $\sin 47 = \cos 43$

$\cos\theta = \sin(90-\theta)$ $\quad$ eg $\cos 88 = \sin 2$

$\tan\theta = \frac{1}{\tan(90-\theta)}$ $\quad$ eg $\tan 70 = \frac{1}{\tan 20}$

* $\tan\theta = \frac{b}{a} = \frac{b/c}{a/c} = \frac{\sin\theta}{\cos\theta}$

* $a^2 + b^2 = c^2$

$(c\cos\theta)^2 + (c\sin\theta)^2 = c^2$

$c^2\cos^2\theta + c^2\sin^2\theta = c^2$

$\cos^2\theta + \sin^2\theta = 1$

Avoid common mistakes like

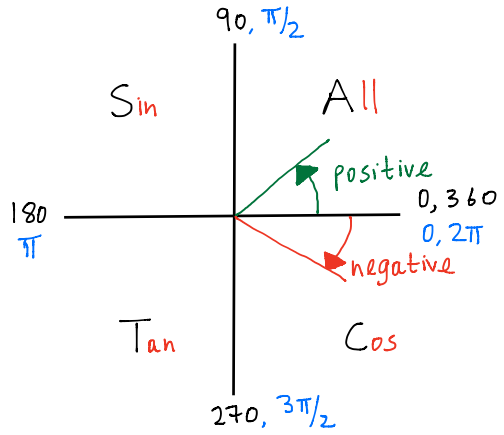
✗ $\sin\theta + \cos\theta = 1$

✗ $(\sin\theta + \cos\theta)^2 = 1$

* Sin, cos and tan of some common angles

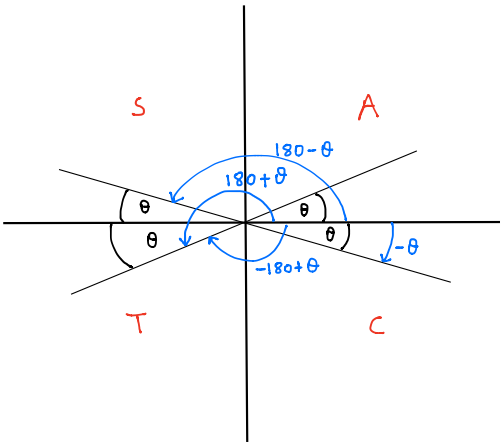
Degrees	0	30	45	60	90
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
Sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
Cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
Tan	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞

*



All Scientists Take Chemistry

* We can derive "nice" relationships between angles in the four quadrants



eg $\sin(180-\theta) = \sin\theta$

$\sin(180+\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

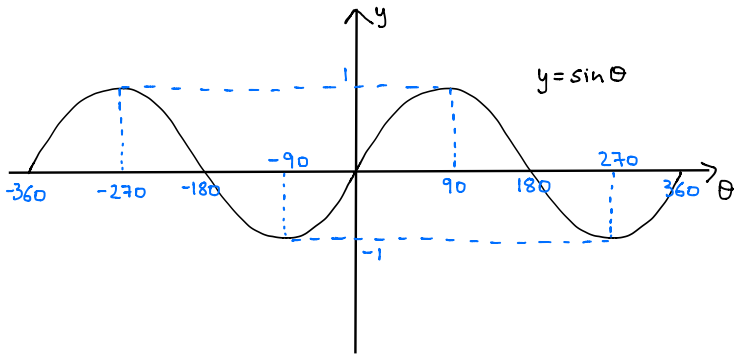
$\cos(-180+\theta) = -\cos\theta$

$\tan(180+\theta) = \tan\theta$

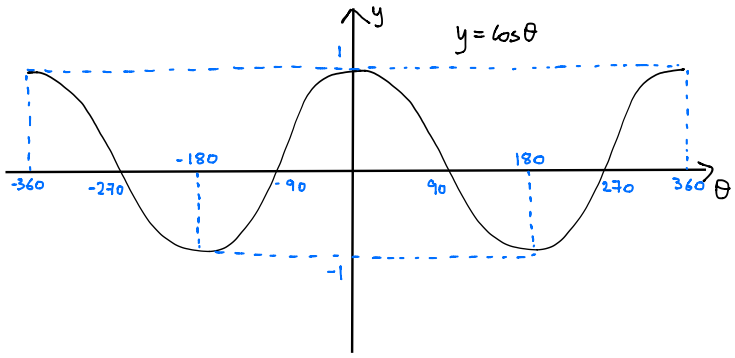
$\tan(180-\theta) = -\tan\theta$

and many others.

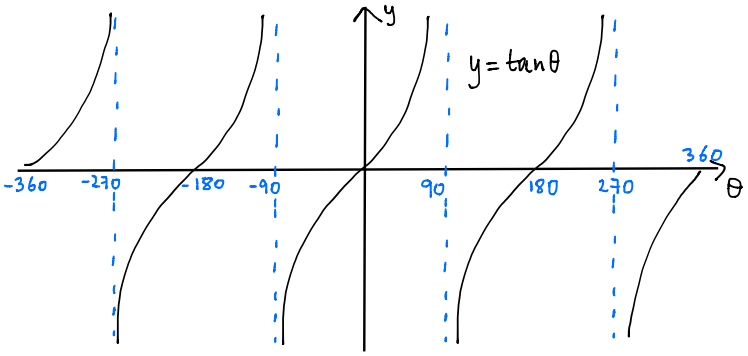
* Graphs of $\sin\theta$, $\cos\theta$, $\tan\theta$



- Maximum: 1
- Minimum: -1
- Cuts on the x-axis at 0, 180, 360 etc
- Period 360° (ie the graph repeats every 360°).



- Maximum : 1
- Minimum : -1
- Cuts on the x-axis at 90, 270 etc
- Period 360° (ie the graph repeats every 360°).



- Cuts on the x-axis at 0, 180, 360 etc
- Period 180° (ie the graph repeats every 180°).
- Asymptotes at $x = 90, 270$

* Transformations of graphs

- $f(x+\alpha)$	Translation α units to the left
- $f(kx)$	Stretch along the horizontal by scale factor $1/k$
- $f(x)+\alpha$	Translation α units up
- $kf(x)$	Stretch along the vertical by scale factor k
- $f(-x)$	Reflection along the y-axis
- $-f(x)$	Reflection along the x-axis

* Solving trigonometric equations (using the formulae)

$$\sin\theta: \begin{cases} \theta = 360n + \alpha & \theta = 2\pi n + \alpha \\ \theta = 360n + 180 - \alpha & \theta = 2\pi n + \pi - \alpha \end{cases}$$

$$\cos\theta: \quad \theta = 360n \pm \alpha \qquad \theta = 2\pi n \pm \alpha$$

$$\tan\theta: \quad \theta = 180n + \alpha \qquad \theta = \pi n + \alpha$$

* If you are asked to prove an identity then start from one side and gradually work your way to the other side of the identity
- Do not work on both sides of the identity at the same time
- Show all workings in every detail.