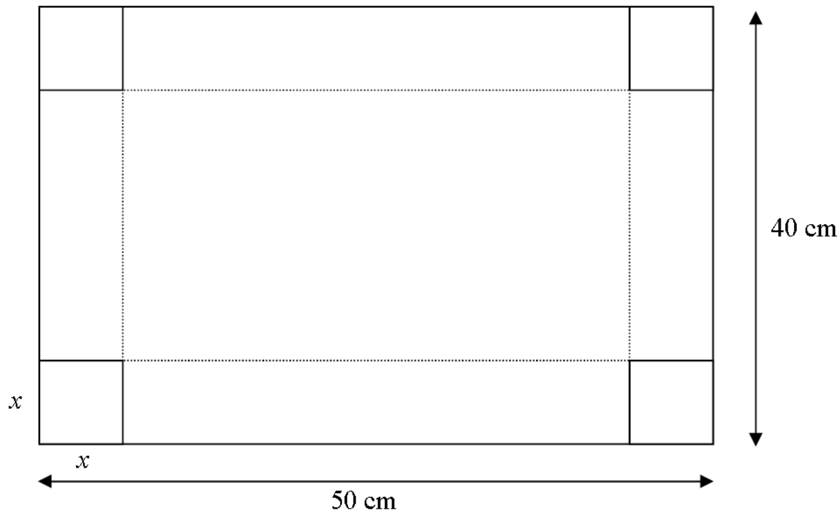
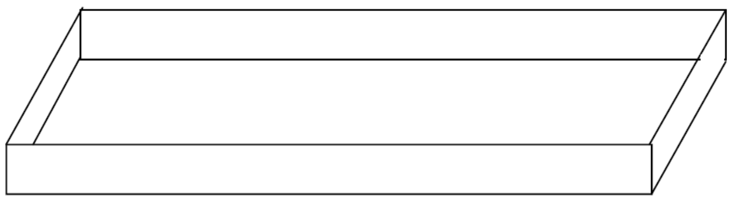
CORE MATHEMATICS 2
CHAPTERS 9 – DIFFERENTIATION AND 11 – INTEGRATION
EXTRA PRACTICE

1. (i) Differentiate $2x^3 + \sqrt{x} + \dfrac{x^2 + 2x}{x^2}$ with respect to x

(ii) Evaluate $\displaystyle\int_1^4 \left(\frac{x}{2} + \frac{1}{x^2}\right) \mathrm{d}x$.

2.





A rectangular sheet of metal measures 50 cm by 40 cm. Squares of side x cm are cut from each corner of the sheet and the remainder is folded along the dotted lines to make an open tray, as shown in Fig. 3.

(a) Show that the volume, V cm^3, of the tray is given by $V = 4x(x^2 - 45x + 500)$.

(b) State the range of possible values of x.

(c) Find the value of x for which V is a maximum.

(d) Hence find the maximum value of V.

(e) Justify that the value of V you found in part (d) is a maximum.

3.

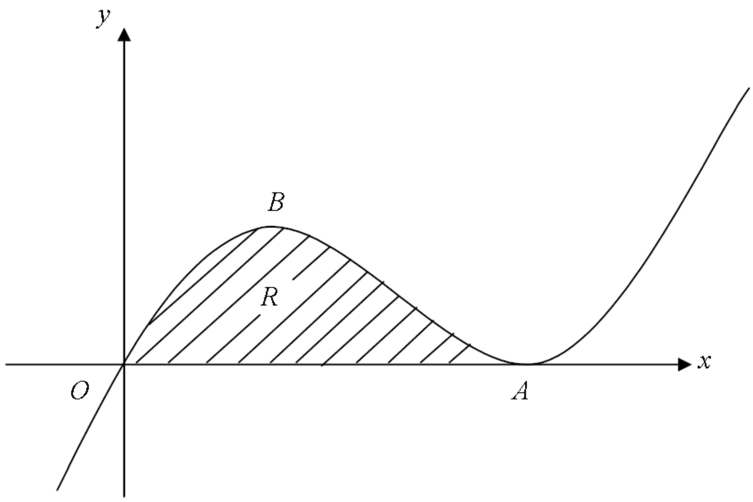


Fig. 2 shows part of the curve with equation $y = x^3 - 6x^2 + 9x$. The curve touches the x-axis at A and has a maximum turning point at B.

(a) Show that the equation of the curve may be written as $y = x(x-3)^2$, and hence write down the coordinates of A.

(b) Find the coordinates of B.

The shaded region R is bounded by the curve and the x-axis.

(c) Find the area of R.

4.

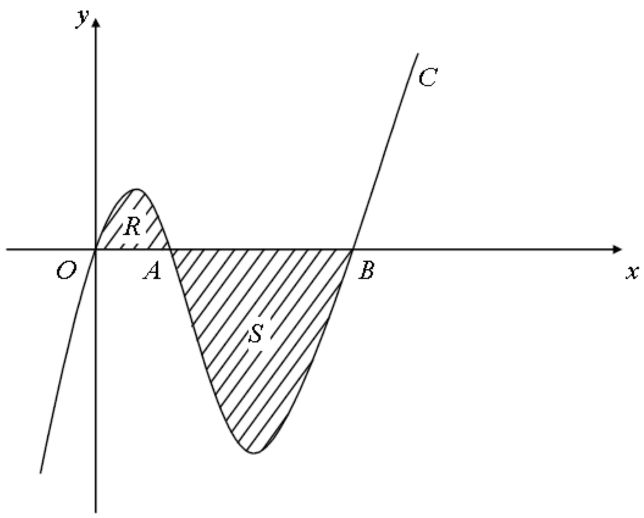


Fig. 2 shows part of the curve C with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 - 6x^2 + 5x$.
The curve crosses the x-axis at the origin O and at the points A and B.

(a) Factorise $\mathrm{f}(x)$ completely.

(b) Write down the x-coordinates of the points A and B.

(c) Find the gradient of C at A.

The region R is bounded by C and the line OA, and the region S is bounded by C and the line AB.

(d) Use integration to find the area of the combined regions R and S, shown shaded in Fig.2.

5.

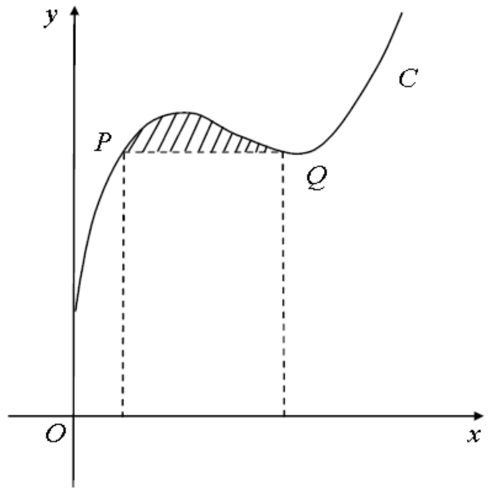


Fig. 3 shows a sketch of part of the curve C with equation $y = x^3 - 7x^2 + 15x + 3, \quad x \geq 0$.

The point P, on C, has x-coordinate 1 and the point Q is the minimum turning point of C.

(a) Find $\frac{dy}{dx}$.

(b) Find the coordinates of Q.

(c) Show that PQ is parallel to the x-axis.

(d) Calculate the area, shown shaded in Fig. 3, bounded by C and the line PQ.

6.

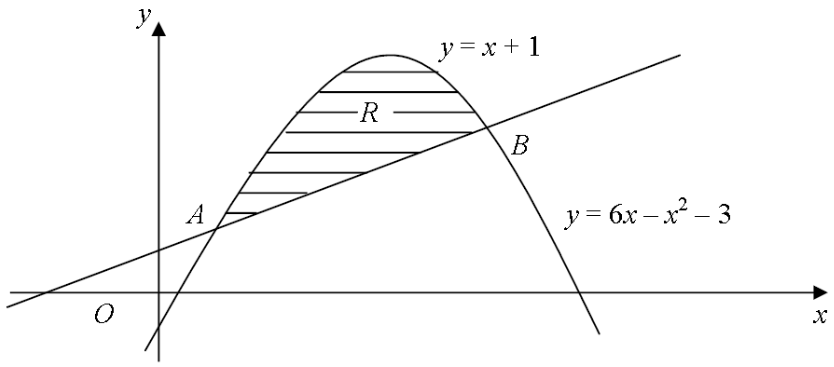


Fig. 2 shows the line with equation $y = x + 1$ and the curve with equation $y = 6x - x^2 - 3$.

The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

The shaded region R is bounded by the line and the curve.

(b) Calculate the area of R.

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7. For the curve C with equation $y = x^4 - 8x^2 + 3$,

(a) find $\frac{dy}{dx}$,

(b) find the coordinates of each of the stationary points,

(c) determine the nature of each stationary point.

The point A, on the curve C, has x-coordinate 1.

(d) Find an equation for the normal to C at A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

8.

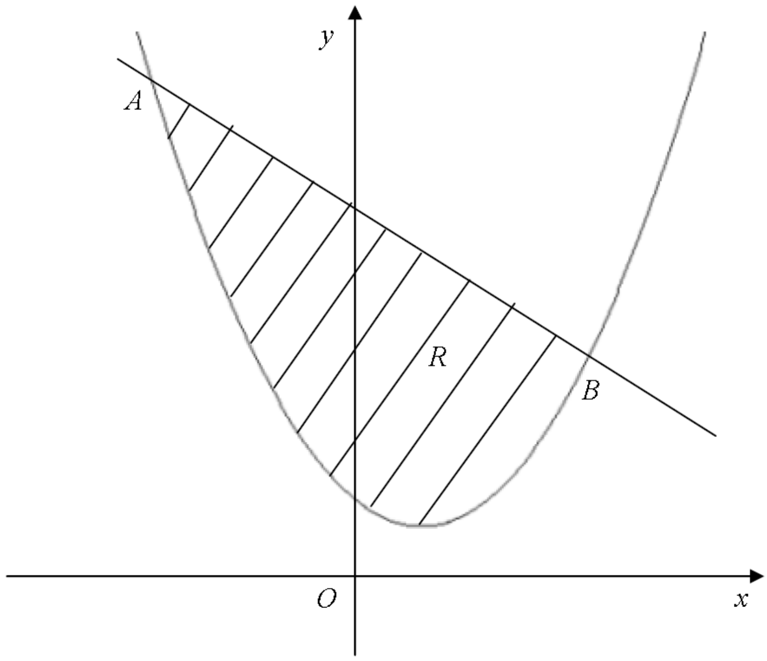

Fig. 3 shows the line with equation $y = 9 - x$ and the curve with equation $y = x^2 - 2x + 3$.
The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

The shaded region R is bounded by the line and the curve.

(b) Calculate the area of R.

9.

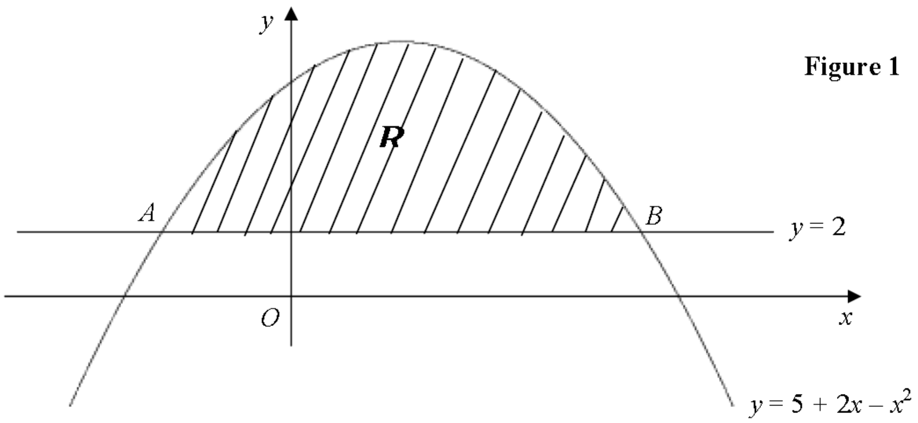

Figure 1

Fig. 1 shows the curve with equation $y = 5 + 2x - x^2$ and the line with equation $y = 2$. The curve and the line intersect at the points A and B.

(a) Find the x-coordinates of A and B.

The shaded region R is bounded by the curve and the line.

(b) Find the area of R.

10.

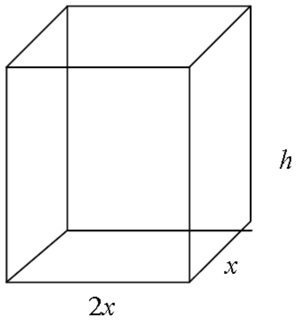

Figure 3

A manufacturer produces cartons for fruit juice. Each carton is in the shape of a closed cuboid with base dimensions $2x$ cm by x cm and height h cm, as shown in Fig. 3.

Given that the capacity of a carton has to be 1030 cm^3,

(*a*) express h in terms of x,

(*b*) show that the surface area, A cm^2, of a carton is given by $A = 4x^2 + \dfrac{3090}{x}$.

The manufacturer needs to minimise the surface area of a carton.

(*c*) Use calculus to find the value of x for which A is a minimum.

(*d*) Calculate the minimum value of A.

(*e*) Prove that this value of A is a minimum.

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11. On a journey, the average speed of a car is v m s^{-1}. For $v \geq 5$, the cost per kilometre, C pence, of the journey is modelled by $C = \dfrac{160}{v} + \dfrac{v^2}{100}$.

Using this model,

(*a*) show, by calculus, that there is a value of v for which C has a stationary value, and find this value of v.

(*b*) Justify that this value of v gives a minimum value of C.

(*c*) Find the minimum value of C and hence find the minimum cost of a 250 km car journey.

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12. A pencil holder is in the shape of an open circular cylinder of radius r cm and height h cm. The surface area of the cylinder (including the base) is 250 cm^2.

(a) Show that the volume, V cm^3, of the cylinder is given by $V = 125r - \dfrac{\pi r^3}{2}$.

(b) Use calculus to find the value of r for which V has a stationary value.

(c) Prove that the value of r you found in part (b) gives a maximum value for V.

(d) Calculate, to the nearest cm^3, the maximum volume of the pencil holder.

13.

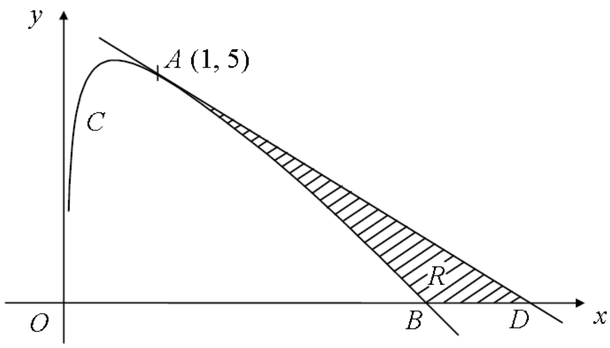

Figure 2 shows part of the curve C with equation

$$y = 9 - 2x - \frac{2}{\sqrt{x}}, \qquad x > 0.$$

The point $A(1, 5)$ lies on C and the curve crosses the x-axis at $B(b, 0)$, where b is a constant and $b > 0$.

(a) Verify that $b = 4$.

The tangent to C at the point A cuts the x-axis at the point D, as shown in Fig. 2.

(b) Show that an equation of the tangent to C at A is $y + x = 6$.

(c) Find the coordinates of the point D.

The shaded region R, shown in Fig. 2, is bounded by C, the line AD and the x-axis.

(d) Use integration to find the area of R.

14.

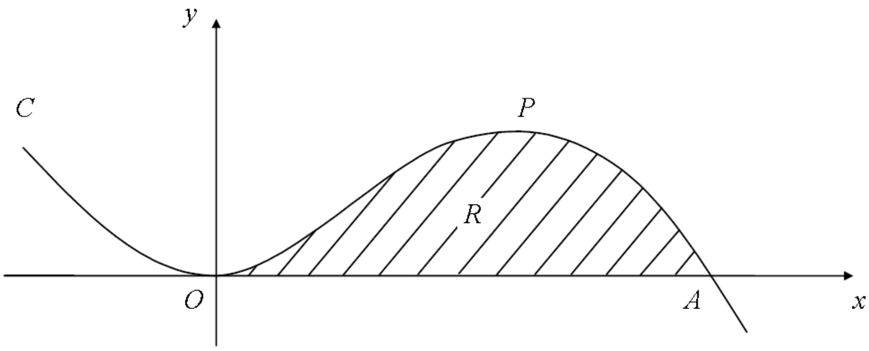

Fig. 1 shows part of the curve C with equation $y = \frac{3}{2}x^2 - \frac{1}{4}x^3$.

The curve C touches the x-axis at the origin and passes through the point $A(p, 0)$.

(*a*) Show that $p = 6$.

(*b*) Find an equation of the tangent to C at A.

The curve C has a maximum at the point P.

(*c*) Find the x-coordinate of P.

The shaded region R, in Fig. 1, is bounded by C and the x-axis.

(*d*) Find the area of R.
