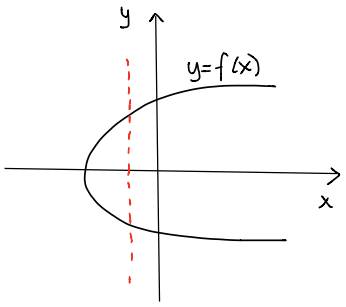
C3 - Chapter 2 - Functions - Summary

* If you are given the graph of a function you can classify it according to the following:

  ① First apply the vertical line rule ie if there is a vertical line that cuts the graph at more than one place then this is a mapping or a one-to-many function.

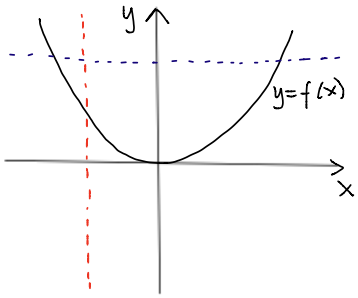
  ② If we pass the vertical line test then apply the horizontal line test ie check if there is a horizontal line that cuts the graph at more than one place. If yes then this is a many-to-one function, if not it is a one-to-one function.

eg



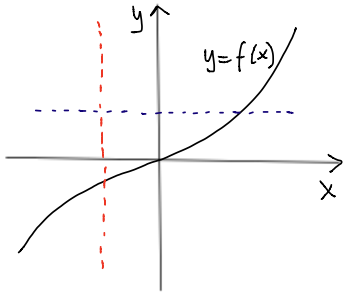
Vertical line test fails

Mapping or a one-to-many function



Vertical line test OK
Horizontal line test fails

One-to-many function



Vertical line test OK
Horizontal line test OK

One-to-one function

* The values that f(x) can take as input are known as the domain. On a graph this corresponds to the x-axis.

* The values that f(x) can output are known as the range. On a graph this corresponds to the y-axis.

* When determining the domain remember that you should never divide by zero and never take the square root of a negative number. Hence, make sure that any "problematic" values of x are excluded.

* Composite functions: $fg(x) = f[g(x)]$ Always start with the function closest to x

* If a function is one-to-one then it has an inverse

To find the inverse $f^{-1}(x)$ of $f(x)$:

① Write it in the form $y = \ldots$

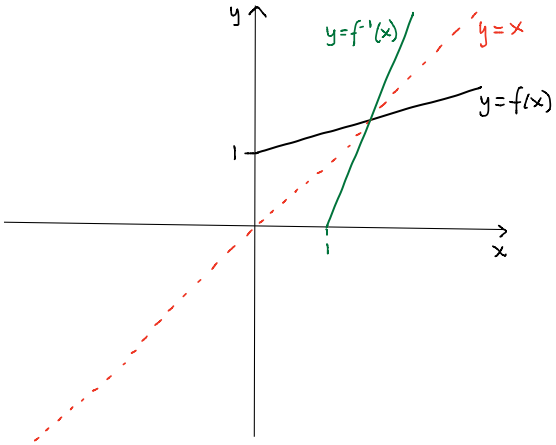
② Interchange x and y

③ Make y the subject of the formula

④ Write it in the form $f^{-1}(x) = \ldots$

On a graph, $f^{-1}(x)$ is the reflection of $f(x)$ along the line $y = x$

eg. $f(x) = \frac{1}{2}x + 1 \quad x \in \mathbb{R}, x \geq 0$



	Domain	Range
$f(x)$	$x \in \mathbb{R}, x \geq 0$	$f(x) \in \mathbb{R}, f(x) \geq 1$
$f^{-1}(x)$	$x \in \mathbb{R}, x \geq 1$	$f^{-1}(x) \in \mathbb{R}, f^{-1}(x) \geq 0$

The domain of $f(x)$ is the range of $f^{-1}(x)$

The range of $f(x)$ is the domain of $f^{-1}(x)$

Note that any point of intersection of $f(x)$ with $f^{-1}(x)$ is also a point of intersection with the mirror line

ie $f(x) = f^{-1}(x) \iff f(x) = x \iff f^{-1}(x) = x$

* We may use the graphical relationship between $f(x)$ and $f^{-1}(x)$ to derive the graph of $y = \sqrt{x}$

If $f(x) = x^2, \quad x \in \mathbb{R}, x \geq 0$ then $f^{-1}(x) = \sqrt{x} \quad x \in \mathbb{R}, x \geq 0$