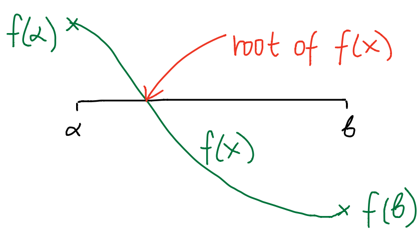
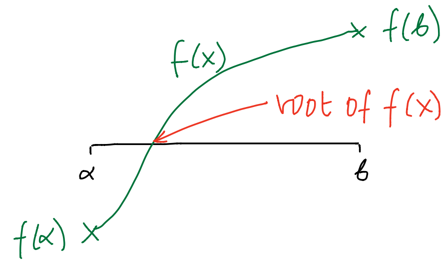
C3 - Chapter 4 - Numerical methods - Summary

* Reminder: A root of $f(x)$ is a solution of the equation $f(x)=0$
* If there is a change of sign between $f(\alpha)$ and $f(b)$, where α and b are constants such that $\alpha<b$, then there is a root of $f(x)$ in the interval (α, b)





$f(\alpha)>0$, $f(b)<0$

Change of sign, therefore a root exists in the interval (α, b)

$f(\alpha)<0$, $f(b)>0$

Change of sign, therefore a root exists in the interval (α, b)

* NOTE: When looking for a change of sign you should always be considering the equation that is equal to zero

EXTENDED EXAMPLE

$$f(x) = x^3+6x^2-18x$$

a) Show that $f(x)$ has a root in the interval $(1.9, 2.4)$

b) Find $f'(x)$ (ie dy/dx)

c) P is a minimum point of $f(x)$. Show that the x-coordinate of P

i) lies between 1 and 2

ii) satisfies the equation $x = \frac{6-x^2}{4}$

d) Use the iteration formula $x_{n+1} = \frac{6-x^2}{4}$ with $x_0 = 1$ to find x_1, x_2 and x_3. Give your answer to 3 dp's.

e) Show that the x-coordinate of P is 1.162 correct to 3 dp's.

a) $f(1.9) = 1.9^3 + 6(1.9^2) - 18(1.9) = -5.681$

$f(2.4) = 2.4^3 + 6(2.4^2) - 18(2.4) = 5.184$

Change of sign therefore a root of $f(x)$ lies in the interval $(5.8, 6.3)$.

b) $f'(x) = 3x^2 + 12x - 18$

c) i) At P $f'(x) = 0$ $\Rightarrow f'(1) = 3(1)^2 + 12(1) - 18 = -3$

$f'(2) = 3(2^2) + 12(2) - 18 = 18$

Change of sign, therefore the x-coordinate of P lies in the interval $(1, 2)$.

ii) $f'(x) = 0 \Rightarrow 3x^2 + 12x - 18 = 0$

$3(x^2 + 4x - 6) = 0$

$x^2 + 4x - 6 = 0$

$x = \frac{6 - x^2}{4}$ AS REQUIRED

d) $x_0 = 1$ $x_1 = 1.25$ $x_2 = 1.109$ $x_3 = 1.192$

e) $x_4 = 1.144592195$ $x_5 = 1.172477177$ $x_6 = 1.156324317$

$x_7 = 1.165728518$ $x_8 = 1.160269255$ $x_9 = 1.163443814$

$x_{10} = 1.161599623$ $x_{11} = 1.162671579$ $x_{12} = 1.1620487$

$x_{13} = 1.162410705$ $x_{14} = 1.162200338$ $x_{15} = 1.162322593$

Since the 3rd decimal place has stabilised the root is 1.162 correct to 3dp's.

Alternatively we could consider a suitable interval.

$f'(1.1625) = 1.406 \times 10^{-3}$ Change of sign, therefore a root exists

$f'(1.1615) = -4.918 \times 10^{-3}$ in the interval $(1.1615, 1.1625)$ which rounds to 1.162 correct to 3 dp's.