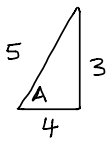
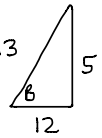
C3 - Chapter 7 - Trigonometry - Practice handout - Solutions

1. $\operatorname{cosec} A = 5/3 \Rightarrow \sin A = 3/5$



$\cos A = 4/5$

$\sin B = 5/13$



S	A
T	C

$\cos B = -12/13$

i) $\sin 2B = 2\sin B \cos B = 2 \cdot \frac{5}{13} \cdot \left(-\frac{12}{13}\right) = -120/169$

ii) $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$= 3/5 \cdot (-12/13) - 4/5\,(5/13) = -56/65$

2. $\dfrac{\tan 2x + \tan 40}{1 - \tan 2x \tan 40} = 1$

$\tan(2x+40) = 1$

$\alpha = 45 \Rightarrow 2x + 40 = 180n + 45$

$x = 90n + 2.5$

$x = 2.5, 92.5, 182.5$

3. $\cos\left(x + \pi/3\right) - \cos\left(x - \pi/3\right) = k$

$\cos x \cos \pi/3 - \sin x \sin \pi/3 - \left\{\cos x \cos \pi/3 + \sin x \sin \pi/3\right\} = k$

$-\sqrt{3} \sin x = k$

4. a) $\cos 2x + 3\sin x = 2$

$1 - 2\sin^2 x + 3\sin x - 2 = 0$

$2\sin^2 x - 3\sin x + 1 = 0$

Let $y = \sin x \Rightarrow 2y^2 - 3y + 1 = 0$

$(2y-1)(y-1) = 0$

$y = 1/2$ OR $y = 1$

$\sin x = 1/2$ $\sin x = 1$

$\alpha = 30$ | $\alpha = 90$

$x = 360n + 30$ | $x = 360n + 90$

$x = 360n + 180 - 30$ | $x = 360n + 180 - 90$

$x = 30, 90, 150$

b) $3\cos(2\theta + 60) - \sin(2\theta - 30) = 0$

$3\cos 2\theta \cos 60 - 3\sin 2\theta \sin 60 - \{\sin 2\theta \cos 30 - \cos 2\theta \sin 30\} = 0$

$\frac{3}{2}\cos 2\theta - \frac{3\sqrt{3}}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta + \frac{1}{2}\cos 2\theta = 0$

$2\cos 2\theta - 2\sqrt{3}\sin 2\theta = 0$

$\frac{2\cos 2\theta}{\cos 2\theta} = 2\sqrt{3}\frac{\sin 2\theta}{\cos 2\theta}$

$1 = \sqrt{3}\tan 2\theta$

$\tan 2\theta = 1/\sqrt{3}$

$\alpha = 30$

$2\theta = 180n + 30$

$\theta = 90n + 15$

$\theta = 15, 105, 195, 285$

5. a) $LHS = \tan x(1 + \cos 2x)$

$= \frac{\sin x}{\cos x}\left(1 + 2\cos^2 x - 1\right)$

$= \frac{2\sin x \cos^2 x}{\cos x} = 2\sin x \cos x = \sin 2x = RHS$

b) $LHS = \frac{2}{1+\cos x} = \frac{2}{1 + 2\cos^2\left(\frac{x}{2}\right) - 1} = \frac{2}{2\cos^2\left(\frac{x}{2}\right)} = \frac{1}{\cos^2\left(\frac{x}{2}\right)} = \sec^2\left(\frac{x}{2}\right) = RHS$

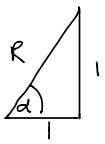
AS REQUIRED

6 a) $\cos x - \sin x = R\cos(x + \alpha)$

$= R\cos x \cos\alpha - R\sin x \sin\alpha$

$\cos x = R\cos x \sin\alpha$ | $\sin x = R\sin x \sin\alpha$

$\frac{1}{R} = \cos\alpha$ | $\frac{1}{R} = \sin\alpha$



$R=\sqrt{1^2+1^2}=\sqrt{2}$ $\quad \tan\alpha=1 \Rightarrow \alpha=\pi/4$

$\cos x-\sin x=\sqrt{2}\cos(x+\pi/4)$

b) Maximum value is $\sqrt{2}$ when $\cos(x+\pi/4)=1$

$\alpha=0$

$x+\pi/4=2\pi n$

$x=2\pi n-\pi/4$

$\therefore x=7\pi/4$

c) $\cos x+\sqrt{2}\cos(3x-\pi/4)=\sin x$

$\cos x-\sin x=-\sqrt{2}\cos(3x-\pi/4)$

$\sqrt{2}\cos(x+\pi/4)=-\sqrt{2}\cos(3x-\pi/4)$

$\cos(x+\pi/4)=-\cos(3x-\pi/4)$

$\cos(3x-\pi/4)+\cos(x+\pi/4)=0$

$0=2\cos\left(\frac{3x-\pi/4+x+\pi/4}{2}\right)\cos\left(\frac{3x-\pi/4-(x+\pi/4)}{2}\right)$

$0=2\cos 2x\cos(x-\pi/4)$

$\cos 2x=0$ $\quad$ OR $\quad$ $\cos(x-\pi/4)=0$

$\alpha=\pi/2$ $\quad$ $\alpha=\pi/2$

$2x=2\pi n\pm\pi/2$ $\quad$ $x-\pi/4=2\pi n\pm\pi/2$

$x=\pi n\pm\pi/4$ $\quad$ $x=2\pi n+\pi/2+\pi/4$ $\quad$ $x=2\pi n-\pi/2+\pi/4$

$x=2\pi n+3\pi/4$ $\quad$ $x=2\pi n-\pi/4$

$x=\pi/4,\ 3\pi/4$

7 a) LHS $=\cos(x+30)+\sin x=\cos x\cos 30-\sin x\sin 30+\sin x$

$=\cos x\cos 30-\frac{1}{2}\sin x+\sin x=\cos x\cos 30+\frac{1}{2}\sin x$

$=\cos x\cos 30+\sin 30\sin x=\cos(x-30)=$ RHS AS REQUIRED

b) $\cos(x+30)+\sin x=\cos(x-30)$

$\cos(x+30)-\cos(x-30)=-\sin x$

$\cos 75-\cos 15=\cos(45+30)-\cos(45-30)$

$=-\sin(45)=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$