Worksheet on Chapter 7 – Further Trigonometry

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1. Given that $\cos ecA = \frac{5}{3}$, $0 < A < 90°$ and that $\sin B = \frac{5}{13}$, $90° < B < 180°$, find the exact value of:

   (i) $\sin 2B$

   (ii) $\sin(A - B)$

2. Find the values of x in the interval $0 < x < 270°$ which satisfy the equation

$$\frac{\tan 2x + \tan 40°}{1 - \tan 2x \tan 40°} = 1.$$

3. Find the value of k such that for all real values of x, $\cos\left(x + \frac{\pi}{3}\right) - \cos\left(x - \frac{\pi}{3}\right) = k$.

4. Solve each equation for x in the interval $0° \le x \le 360°$.

   (a) $\cos 2x + 3\sin x = 2$

   (b) $3\cos(2\theta + 60)° - \sin(2\theta - 30)° = 0$

5. Prove each identity.

   (a) $\tan x(1 + \cos 2x) \equiv \sin 2x$

   (b) $\frac{2}{1 + \cos x} \equiv \sec^2 \frac{x}{2}$

6. (a) Express $\cos x - \sin x$ in the form $R\cos(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

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(b) Find the maximum value of $\cos x - \sin x$ and the smallest positive value of x for which this occurs.

(c) Using the identity

$$\cos X + \cos Y \equiv 2\cos\frac{X+Y}{2}\cos\frac{X-Y}{2}$$

find in terms of π the values of x in the interval $[0, \pi]$ for which

$$\cos x + \sqrt{2}\cos\left(3x - \frac{\pi}{4}\right) = \sin x .$$

7. (a) Prove that for all real values of x, $\cos(x+30)^\circ + \sin x \equiv \cos(x-30)^\circ$.

(b) Hence, find the exact value of $\cos 75^\circ - \cos 15^\circ$, giving your answer in the form $k\sqrt{2}$.