C3 - Chapter 8 - Differentiation - Extra practice handout - Solutions

1a) $y=\sqrt{5x-4} = (5x-4)^{1/2}$ $\qquad \frac{dy}{dx} = \frac{1}{2}(5)(5x-4)^{-1/2}$

b) $y = x^2 \sec 2x$ $\qquad \frac{dy}{dx} = 2x \sec 2x + 2x^2 \sec 2x \tan 2x$

c) $y = \frac{\cos(x^2)}{2x}$ $\qquad \frac{dy}{dx} = \frac{2x\cos(x^2) \cdot 2x - 2\cos(x^2)}{(2x)^2}$

d) $y = (e^{2x}+1)^{1/2}$ $\qquad \frac{dy}{dx} = \frac{1}{2} \cdot 2e^{2x}(e^{2x}+1)^{-1/2}$

e) $y = 5\operatorname{cosec}^2 2x$ $\qquad \frac{dy}{dx} = -5 \cdot 2 \cdot 2 \operatorname{cosec} 2x \operatorname{cosec} 2x \cot 2x$

f) $y = \ln(2x-1)$ $\qquad \frac{dy}{dx} = \frac{2}{2x-1}$

g) $y = x \sin\left(\frac{2x}{3}\right)$ $\qquad \frac{dy}{dx} = \sin\left(\frac{2x}{3}\right) + x \cdot \frac{2}{3}\cos\left(\frac{2x}{3}\right)$

2. $x = \frac{y+1}{3-2y}$ $\qquad \frac{dx}{dy} = \frac{(3-2y)-(-2)(y+1)}{(3-2y)^2} = \frac{5}{(3-2y)^2}$

$$\left.\frac{dx}{dy}\right|_{y=2} = 5$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{5}$$

3. a) $y = e^{2x}\sec x$ $\qquad \frac{dy}{dx} = 2e^{2x}\sec x + e^{2x}\sec x \tan x$

When $x=0$, $y=1$ $\qquad \left.\frac{dy}{dx}\right|_{x=0} = 2e^0 \sec 0 + e^0 \sec 0 \tan 0 = 2$

$$y - 1 = 2(x-0)$$

$$y = 2x+1$$

b) Stationary point $\frac{dy}{dx}=0 \Rightarrow 2e^{2x}\sec x + e^{2x}\sec x \tan x = 0$

$$e^{2x}\sec x\{2 + \tan x\} = 0$$

$e^{2x}=0$ Reject not valid $\quad$ OR $\quad$ $\sec x = 0$ Reject not valid $\quad$ OR $\quad$ $\tan x = -2$, $x = -1.11$

4. $y = x^2 - 5x + 2\ln\left(\frac{x}{3}\right)$

$$\frac{dy}{dx} = 2x - 5 + \frac{2 \cdot 1/3}{x/3} = 2x - 5 + \frac{2}{x}$$

Stationary point $\Rightarrow \frac{dy}{dx} = 0$

$$2x - 5 + \frac{2}{x} = 0$$

$$2x^2 - 5x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$x = 1/2$ <u>OR</u> $x = 2$

$y = -9/4 + 2\ln\left(\frac{1}{6}\right)$ $\quad$ $y = -6 + 2\ln(2/3)$

$\therefore \left(1/2,\ -9/4 + 2\ln\left(\frac{1}{6}\right)\right)$ $\quad$ $\left(2,\ -6 + 2\ln(2/3)\right)$