The G C School of Careers
Department of Mathematics

## Homework Assignment On VECTORS

1. The vector equations of the lines $L_{1}$ and $L_{2}$ are given by:

$$
\begin{aligned}
L_{1}: \quad \boldsymbol{r} & =\boldsymbol{i}+3 \boldsymbol{j}+5 \boldsymbol{k}+\lambda(\mathbf{i}+2 \boldsymbol{j}-\boldsymbol{k}) \\
L_{2}: & \boldsymbol{r}=-2 \boldsymbol{i}+3 \boldsymbol{j}-4 \boldsymbol{k}+\mu(2 \boldsymbol{i}+\boldsymbol{j}+4 \boldsymbol{k}),
\end{aligned}
$$

where $\lambda$ and $\mu$ are parameters.
(a) Show that $L_{1}$ and $L_{2}$ intersect and find the coordinates of $B$, their point of intersection.
(b) Show that $L_{1}$ is perpendicular to $L_{2}$.

The point $A$ with $x$-coordinate 4 lies on $L_{1}$ and the point $C$ with $x$-coordinate 6 lies on $L_{2}$.
(c) Find, in its simplest form, the exact area of the triangle $A B C$.
2. The points $A$ and $B$ have coordinates $(3,9,-7)$ and $(13,-6,-2)$ respectively.
(a) Find, in vector form, an equation for the line $l$ which passes through $A$ and $B$.
(b) Show that the point $C$ with coordinates $(9,0,-4)$ lies on $l$.

The point $D$ is the point on $l$ closest to the origin, $O$.
(c) Find the coordinates of $D$.
(d) Find the area of triangle $O A B$ to 3 significant figures.
3. Referred to an origin $O$, the points $A$ and $B$ have position vectors given by:

$$
\begin{aligned}
& \overrightarrow{O A}=7 \boldsymbol{i}+3 \boldsymbol{j}+8 \boldsymbol{k} \\
& \overrightarrow{O B}=5 \boldsymbol{i}+4 \boldsymbol{j}+6 \boldsymbol{k}
\end{aligned}
$$

(a) Show that the point $P$ with position vector given by:

$$
\overrightarrow{O P}=(5-2 \lambda) \boldsymbol{i}+(4+\lambda) \boldsymbol{j}+(6-2 \lambda) \boldsymbol{k}
$$

where $\lambda$ is a parameter, lies on the straight line $L$ passing through the points $A$ and $B$.
(b) Find the value of $\lambda$ for which $O P$ is perpendicular to $L$.

With centre $O$ and radius $O A$, a circle is drawn to cut the line $L$ at the points $A$ and $C$.
(c) Determine the position vector of $C$.
4. A line $l_{1}$ passes through the point $A$, with position vector $5 \boldsymbol{i}+3 \boldsymbol{j}$, and the point $B$, with position vector $-2 \boldsymbol{i}-4 \boldsymbol{j}+7 \boldsymbol{k}$.
(a) Write down an equation of the line $l_{1}$.

A second line $l_{2}$ has equation $\boldsymbol{r}=\mathbf{i}-3 \mathbf{j}-4 \boldsymbol{k}+\mu(\mathbf{i}+2 \boldsymbol{j}+3 \boldsymbol{k})$, where $\mu$ is a parameter.
(b) Show that $l_{1}$ and $l_{2}$ are perpendicular to each other.
(c) Show that the two lines meet, and find the position vector of the point of intersection.

The point $C$ has position vector $2 \mathbf{i}-\boldsymbol{j}-\boldsymbol{k}$.
(d) Show that $C$ lies on $l_{2}$.

The point $D$ is the image of $C$ after reflection in the line $l_{1}$.
(e) Find the position vector of $D$.

