HOMEWORK ASSIGNMENT ON VECTORS

1a)

$$L_1 = \begin{pmatrix} 1+\lambda \\ 3+2\lambda \\ 5-\lambda \end{pmatrix} \qquad L_2 = \begin{pmatrix} -2+2\mu \\ 3+\mu \\ -4+4\mu \end{pmatrix}$$

$1+\lambda = -2+2\mu$

$\lambda = 2\mu - 3$

$3+2\lambda = 3+\mu$

$2\lambda = \mu$

$5-\lambda = -4+4\mu$

Substitute $\lambda=1, \mu=2$

$\lambda = 4\lambda - 3$

$3 = 3\lambda$

$\lambda = 1$

$\mu = 2$

$5-1 = 4$

$-4+4\cdot2 = 4$

$\therefore$ L_1 and L_2 intersect

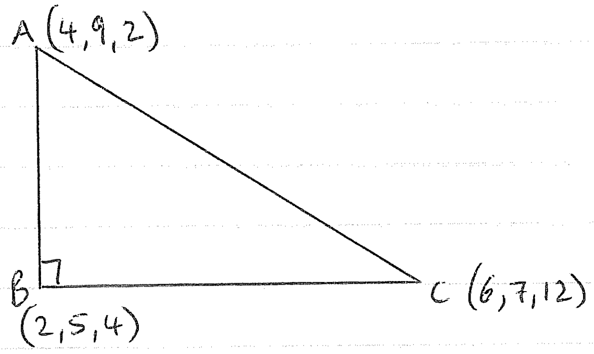
B has coordinates $(2,5,4)$

b) $(\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}) \cdot (2\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k}) = 2+2-4=0$

$\therefore$ L_1 and L_2 are perpendicular to each other.

c) A has coordinates $(4,9,2)$

C has coordinates $(6,7,12)$



$AB = \sqrt{(4-2)^2 + (9-5)^2 + (2-4)^2} = \sqrt{24}$

$BC = \sqrt{(6-2)^2 + (7-5)^2 + (12-4)^2} = \sqrt{84}$

$\therefore$ Area of triangle ABC $= \dfrac{\sqrt{24} \times \sqrt{84}}{2}$

$= 6\sqrt{14}$ sq units.

2a) $\overrightarrow{AB} = \begin{pmatrix} 13 \\ -6 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} = \begin{pmatrix} 10 \\ -15 \\ 5 \end{pmatrix}$

l: $\underset{\sim}{r} = 3\underset{\sim}{i} + 9\underset{\sim}{j} - 7\underset{\sim}{k} + \lambda(10\underset{\sim}{i} - 15\underset{\sim}{j} + 5\underset{\sim}{k})$

b) $\begin{pmatrix} 3+10\lambda \\ 9-15\lambda \\ -7+5\lambda \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ -4 \end{pmatrix}$

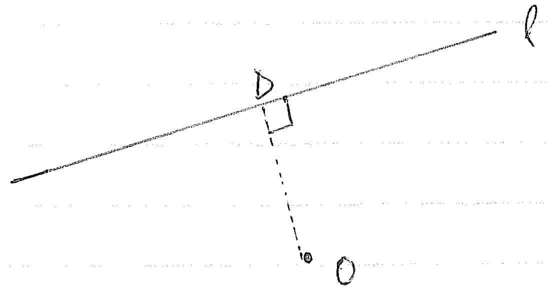
$3+10\lambda = 9 \Rightarrow \lambda = 3/5$

$9-15\lambda = 0 \Rightarrow \lambda = 3/5$

$-7+5\lambda = -4 \Rightarrow \lambda = 3/5$

$\therefore$ C $(9,0,-4)$ lies on l. AS REQUIRED.

c) Since D lies on l, its coordinates are $(3+10\lambda, 9-15\lambda, -7+5\lambda)$



Shortest distance from the line l to the origin is the perpendicular distance

$\Rightarrow \overrightarrow{OD} \cdot \overrightarrow{AB} = 0$

$\begin{pmatrix} 10 \\ -15 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 3+10\lambda \\ 9-15\lambda \\ -7+5\lambda \end{pmatrix} = 0$

$10(3+10\lambda) - 15(9-15\lambda) + 5(-7+5\lambda) = 0.$

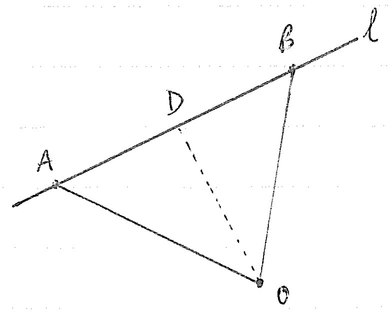
$30 + 100\lambda - 135 + 225\lambda - 35 + 25\lambda = 0$

$350\lambda = 140$

$\lambda = 2/5$

$\therefore$ D has coordinates $(7, 3, -5)$

d)



$$|\vec{AB}| = \sqrt{6^2+15^2+5^2} = \sqrt{350}$$

$$|\vec{OD}| = \sqrt{7^2+3^2+(-5)^2} = \sqrt{83}$$

$$\text{Area of triangle} = \frac{\sqrt{350} \times \sqrt{83}}{2}$$

$$= 85.2 \text{ sq units}$$

3 a) $\vec{OP} = (5-2\lambda)\underset{\sim}{i} + (4+\lambda)\underset{\sim}{j} + (6-2\lambda)\underset{\sim}{k}$

$$= 5\underset{\sim}{i} + 4\underset{\sim}{j} + 6\underset{\sim}{k} + \lambda(-2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k})$$

$$\vec{AB} = \vec{OB} - \vec{OA} = (5\underset{\sim}{i} + 4\underset{\sim}{j} + 6\underset{\sim}{k}) - (7\underset{\sim}{i} + 3\underset{\sim}{j} + 8\underset{\sim}{k})$$

$$= -2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$$

$$\Rightarrow \vec{OP} = \vec{OB} + \lambda\vec{AB}$$

Hence P lies on line L.

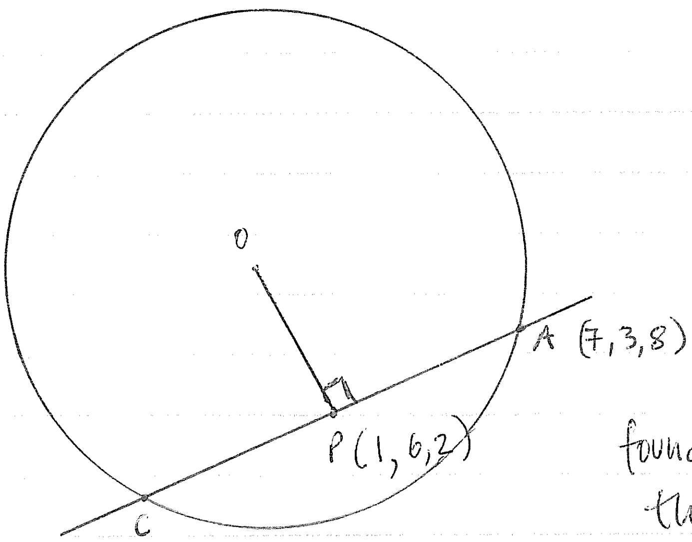
b) $\vec{OP} \cdot \vec{AB} = \begin{pmatrix} 5-2\lambda \\ 4+\lambda \\ 6-2\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} = -2(5-2\lambda) + 4+\lambda - 2(6-2\lambda)$

$$= -10 + 4\lambda + 4 + \lambda - 12 + 4\lambda = 0$$

$$\Rightarrow -18 + 9\lambda = 0$$

$$\lambda = 2$$

c)



found by setting $\lambda=2$ into the expression for $\vec{OP}$.

Now, $\vec{OC} = \vec{OP} + \vec{PC}$

$$= \vec{OP} + \vec{AP}$$

$$= \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} + \begin{pmatrix} -6 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} -5 \\ 9 \\ -4 \end{pmatrix}$$

4\. a) $\vec{AB} = \begin{pmatrix} -2 \\ -4 \\ 7 \end{pmatrix} - \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ 7 \end{pmatrix}$

$$\Rightarrow \quad l_1: \quad \underset{\sim}{r} = 5\underset{\sim}{i} + 3\underset{\sim}{j} + \lambda(-7\underset{\sim}{i} - 7\underset{\sim}{j} + 7\underset{\sim}{k})$$

b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ -7 \\ 7 \end{pmatrix} = -7 - 14 + 21 = 0$

$\therefore$ l_1 and l_2 are perpendicular to each other.

c) $\begin{pmatrix} 5-7\lambda \\ 3-7\lambda \\ 7\lambda \end{pmatrix} = \begin{pmatrix} 1+\mu \\ -3+2\mu \\ -4+3\mu \end{pmatrix}$

$5-7\lambda = 1+\mu$ [1]

$3-7\lambda = -3+2\mu$ [2]

$7\lambda = -4+3\mu$ [3]

[1] - [2] $\Rightarrow$ $2 = 4-\mu$ $\Rightarrow$ $\mu = 2$

Substitute $\mu=2$ into [1] $\Rightarrow$ $5-7\lambda = 1+2$

$$2 = 7\lambda$$

$$\lambda = 2/7$$

Check in equation [3] $\quad 7\lambda = 7 \cdot 2/7 = 2$

$$-4+3\mu = -4+3\cdot 2 = 2$$

$\therefore$ The two lines meet

The position vector of the point of intersection is

$$3\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$$

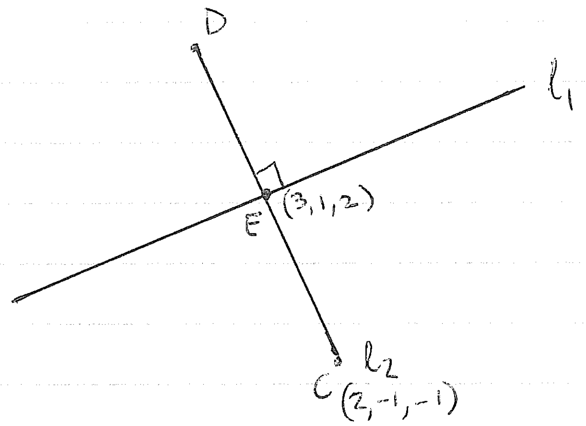
d) $\begin{pmatrix} 1+\mu \\ -3+2\mu \\ -4+3\mu \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$ $\quad 1+\mu = 2 \quad \mu = 1$

$-3+2\mu = -1 \quad \mu = 1$

$-4+3\mu = -1 \quad \mu = 1$

$\therefore$ Point C lies on l_2.

e)



$$\vec{OD} = \vec{OE} + \vec{ED}$$
$$= \vec{OE} + \vec{CE}$$
$$= \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix} = 4\underset{\sim}{i} + 3\underset{\sim}{j} + 5\underset{\sim}{k}$$