C2 - Chapter 6 - Integration - Extra practice - Solutions

a) $\int 3(x+5)^4\,dx = \frac{3(x+5)^5}{5} + C$

b) $\int \frac{4}{(x-1)(x+1)^2}\,dx = \int \frac{1}{x-1} - \frac{1}{x+1} - 2(x+1)^{-2}\,dx$

$\frac{4}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$= \ln|x-1| - \ln|x+1| - \frac{2(x+1)^{-1}}{-1} + C$

$4 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x=1 \Rightarrow 4 = 4A \Rightarrow A=1$

$x=-1 \Rightarrow 4 = -2C \Rightarrow C=-2$

$x=0 \Rightarrow 4 = A - B - C$

$4 = 1 - B - (-2) \Rightarrow B = -1$

$\frac{4}{(x-1)(x+1)^2} = \frac{1}{x-1} + \frac{1}{x+1} + \frac{-2}{(x+1)^2}$

c) $\int \frac{8}{3-7x}\,dx = 8\frac{\ln|3-7x|}{-7} + C$

d) $\int \frac{5-4\cos(x/3)}{\sin^2(x/3)}\,dx = \int \frac{5}{\sin^2(x/3)} - \frac{4\cos(x/3)}{\sin^2(x/3)}\,dx = \int 5\operatorname{cosec}^2(x/3) - 4\frac{\cos(x/3)}{\sin(x/3)} \cdot \frac{1}{\sin(x/3)}\,dx$

$= \int 5\operatorname{cosec}^2(x/3) - 4\cot(x/3)\operatorname{cosec}(x/3)\,dx$

$= -\frac{5\cot(x/3)}{1/3} + \frac{4\operatorname{cosec}(x/3)}{1/3} + C$

e) $\int \frac{4}{(5-2x)^3}\,dx = \int 4(5-2x)^{-3}\,dx = \frac{4(5-2x)^{-2}}{(-2)(-2)} + C$

f) $\int \sin 2x\, e^{3\cos 2x}\,dx = \int -\frac{1}{2}e^{3u}\,du = -\frac{1}{2}\frac{e^{3u}}{3} + C$

$= -\frac{1}{2}\frac{e^{3\cos 2x}}{3} + C$

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx$

g) $\int e^{2x} + 3\sin 5x\,dx = \frac{1}{2}e^{2x} - 3\frac{\cos 5x}{5} + C$

h) $\int \frac{x^2}{x^2-25}\,dx = \int 1 + \frac{25}{(x-5)(x+5)}\,dx$

$= \int 1 + \frac{5/2}{x-5} - \frac{5/2}{x+5}\,dx$

$= x + \frac{5}{2}\ln|x-5| - \frac{5}{2}\ln|x+5| + C$

$x^2+0x+0 \,\big|\, x^2-25$
$-\;x^2 \quad -25 \quad 1$
25

$\frac{25}{(x-5)(x+5)} = \frac{A}{x-5} + \frac{B}{x+5}$

$25 = A(x+5) + B(x-5)$

$x=5 \Rightarrow 25 = 10A \Rightarrow A = 5/2$

$x=-5 \Rightarrow 25 = -10B \Rightarrow B = -5/2$

$\frac{25}{(x-5)(x+5)} = \frac{5/2}{x-5} - \frac{5/2}{x+5}$

i) $\int (\cos x - 3)^2\,dx = \int \cos^2 x - 6\cos x + 9\,dx$

$= \int \frac{\cos 2x + 1}{2} - 6\cos x + 9\,dx$

$= \frac{\frac{1}{2}\sin 2x + x}{2} - 6\sin x + 9x + C$

$\cos 2x = 2\cos^2 x - 1$
$\cos^2 x = \frac{\cos 2x + 1}{2}$

j) $\int 3\sin 4x \cos^4 4x\,dx = \int -\frac{3}{4}u^4\,du = -\frac{3}{4}\frac{u^5}{5} + C$

$= -\frac{3}{4}\frac{\cos^5 4x}{5} + C$

$u = \cos 4x$
$du = -4\sin 4x\,dx$
$-\frac{1}{4}du = \sin 4x\,dx$

k) $\int (\sin x\cos x)^2\,dx = \int \left(\frac{1}{2}\sin 2x\right)^2 dx = \int \frac{1}{4}\sin^2 2x$

$= \int \frac{1}{4}\,\frac{1-\cos 4x}{2}\,dx = \frac{1}{4}\left(\frac{x - \frac{1}{4}\sin 4x}{2}\right) + C$

$\cos 2x = 1 - 2\sin^2 x$
$\sin^2 x = \frac{1-\cos 2x}{2}$

$\sin^2 2x = \frac{1-\cos 4x}{2}$

l) $\int \frac{3x-2}{3x^2-4x+2}\,dx = \int \frac{1}{2}\cdot\frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|3x^2-4x+2| + C$

$u = 3x^2-4x+2$
$du = (6x-4)\,dx$
$du = 2(3x-2)\,dx$
$\frac{1}{2}du = (3x-2)\,dx$

m) $\int (\cot x + \operatorname{cosec} x)^2\,dx = \int \cot^2 x + 2\cot x\operatorname{cosec} x + \operatorname{cosec}^2 x\,dx$

$= \int \operatorname{cosec}^2 x - 1 + 2\cot x\operatorname{cosec} x + \operatorname{cosec}^2 x\,dx$

$= -\cot x - x - 2\operatorname{cosec} x - \cot x + C$

n) $\int \frac{2\sin x}{5+2\cos x}\,dx = \int -\frac{1}{2} \cdot \frac{2}{u}\,du = \int -\frac{1}{u}\,du$

$u = 5+2\cos x$
$du = -2\sin x\,dx$
$-\frac{1}{2}\,du = \sin x\,dx$

$= -\ln|u| + C = -\ln|5+2\cos x| + C$

o) $\int \sin 2x \cos 5x\,dx = \int \cos 5x \sin 2x\,dx$

$= \int \frac{1}{2}\left(\sin 7x - \sin 3x\right)dx$

$= \frac{1}{2}\left(-\frac{1}{7}\cos 7x + \frac{1}{3}\cos 3x\right) + C$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

$\frac{A+B}{2} = 5x \Rightarrow A+B = 10x$

$\frac{A-B}{2} = 2x \Rightarrow A-B = 4x$

$A = 7x \quad B = 3x$

p) $\int x(x^2+7)^4\,dx = \int \frac{1}{2}u^4\,du = \frac{1}{2}\frac{u^5}{5} + C$

$= \frac{1}{2}\frac{(x^2+7)^5}{5} + C$

$u = x^2+7$
$du = 2x\,dx$
$\frac{1}{2}\,du = x\,dx$

q) $\int \cos 7x \cos 3x\,dx = \int \frac{1}{2}\left(\cos 10x + \cos 4x\right)dx$

$= \frac{1}{2}\left(\frac{\sin 10x}{10} + \frac{\sin 4x}{4}\right) + C$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

$\frac{A+B}{2} = 7x \Rightarrow A+B = 14x$

$\frac{A-B}{2} = 3x \Rightarrow A-B = 6x$

$A = 10x \quad B = 4x$

r) $\int \operatorname{cosec}^2 x\,(3-\cot x)^2\,dx = \int u^2\,du = \frac{u^3}{3} + C = \frac{(3-\cot x)^3}{3} + C$

$u = 3-\cot x$
$du = \operatorname{cosec}^2 x\,dx$

s) $\int \cos^2 4x\,dx = \int \frac{\cos 8x + 1}{2}\,dx = \frac{1/8\,\sin 8x + x}{2} + C$

$\cos 2x = 2\cos^2 x - 1$
$\cos^2 x = \frac{\cos 2x + 1}{2}$
$\cos^2 4x = \frac{\cos 8x + 1}{2}$

t) $\int \sin 2x\, e^{3\cos 2x}\,dx = \int -\frac{1}{2}e^{3u}\,du = -\frac{1}{2}\frac{e^{3u}}{3} + C$

$= -\frac{1}{2}\frac{e^{3\cos 2x}}{3} + C$

$u = \cos 2x$
$du = -2\sin 2x\,dx$
$-\frac{1}{2}\,du = \sin 2x\,dx$

u) $\int (1-2\sin x)^2\,dx = \int 1 - 4\sin x + 4\sin^2 x\,dx$

$= \int 1 - 4\sin x + 2 - 2\cos 2x\,dx$

$= x + 4\cos x + 2x - \frac{2}{2}\sin 2x + C$

$\cos 2x = 1 - 2\sin^2 x$
$2\sin^2 x = 1 - \cos 2x$
$4\sin^2 x = 2 - 2\cos 2x$

v) $\int \frac{4\sin^2 x - \cos x}{\sin^2 x}\,dx = \int \frac{4\sin^2 x}{\sin^2 x} - \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}\,dx = \int 4 - \cot x \operatorname{cosec} x\,dx$

$= 4x + \operatorname{cosec} x + C$

w) $\int 4e^{-2x} + \frac{1}{x+3} - \cot^2(5x)\,dx = \int 4e^{-2x} + \frac{1}{x+3} - (\operatorname{cosec}^2 5x - 1)\,dx$

$= \int \left(4e^{-2x} + \frac{1}{x+3} - \operatorname{cosec}^2 5x + 1\right) dx$

$= \frac{4e^{-2x}}{-2} + \ln|x+3| + \frac{\cot 5x}{5} + x + C$