INTEGRATION

FORMULAE

$\int x^n \; dx = \frac{x^{n+1}}{n+1} + C$	$\int (ax+b)^n \; dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C$
$\int \frac{1}{x} \; dx = \ln\|x\| + C$	$\int \frac{1}{ax+b} \; dx = \frac{1}{a}\ln\|ax+b\| + C$
$\int e^x \; dx = e^x + C$	$\int e^{ax+b} \; dx = \frac{1}{a}e^{ax+b} + C$
$\int \cos(x) \; dx = \sin(x) + C$	$\int \cos(ax+b) \; dx = \frac{1}{a}\sin(ax+b) + C$
$\int \sin(x) \; dx = -\cos(x) + C$	$\int \sin(ax+b) \; dx = -\frac{1}{a}\cos(ax+b) + C$
$\int \sec^2(x) \; dx = \tan(x) + C$	$\int \sec^2(ax+b) \; dx = \frac{1}{a}\tan(ax+b) + C$
$\int \cos ec^2(x) \; dx = -\cot(x) + C$	$\int \cos ec^2(ax+b) \; dx = -\frac{1}{a}\cot(ax+b) + C$
$\int \sec(x)\tan(x) \; dx = \sec(x) + C$	$\int \sec(ax+b)\tan(ax+b) \; dx = \frac{1}{a}\sec(ax+b) + C$
$\int \cos ec(x)\cot(x) \; dx = -\cos ec(x) + C$	$\int \cos ec(ax+b)\cot(ax+b) \; dx = -\frac{1}{a}\cos sec(ax+b) + C$

$\int \; [f(x)]^n f'(x) \; dx = \frac{f(x)^{n+1}}{n+1} + C$
$\int \; \frac{f'(x)}{f(x)} \; dx = \ln\|f(x)\| + C$
$\int \; e^{f(x)} f'(x) \; dx = e^{f(x)} + C$

*	$\int \tan(x) \; dx = \ln\|\sec(x)\| + C$
*	$\int \cot(x) \; dx = \ln\|\sin(x)\| + C$
*	$\int \sec(x) \; dx = \ln\|\sec(x) + \tan(x)\| + C$
*	$\int \cos ec(x) \; dx = -\ln\|\cos ec(x) + \cot(x)\| + C$

INTEGRATION BY PARTS

* $\int \; u\frac{dv}{dx} \; dx = uv - \int v\frac{du}{dx}dx$

TRAPEZIUM RULE

* $\int_a^b y \; dx \approx \frac{1}{2}h\left[y_o + 2(y_1 + y_2 + ... + y_{n-1}) + y_n\right]$ where $h = \frac{b-a}{n}$

* Given in the formula booklet

AREA AND VOLUME

For cartesian equations:

Area = $\int_a^b y \ dx$	Volume = $\pi \int_a^b y^2 \ dx$

where *a* and *b* are the limits of *x*.

For parametric equations:

Area = $\int_n^m y \ \frac{dx}{dt} \ dt$	Volume = $\pi \int_n^m y^2 \ \frac{dx}{dt} \ dt$

where *m* and *n* are the limits of *t*.

GENERAL TIPS

- When integrating trigonometric terms:

  $\int \sin^2(x) \ dx$: use the identity of $\cos(2x) = 1 - 2\sin^2(x)$
  $\int \cos^2(x) \ dx$: use the identity of $\cos(2x) = 2\cos^2(x) - 1$
  $\int \tan^2(x) \ dx$: use the identity of $\sec^2(x) = 1 + \tan^2(x)$
  $\int \cot^2(x) \ dx$: use the identity of $\cos ec^2(x) = 1 + \cot^2(x)$

- Use the identity $\sin(2x) = 2\sin(x)\cos(x)$ when the function is a product of sin(*x*) and cos(*x*) either in the numerator or in the denominator.

- Products in the form $\sin(A)\cos(B)$ <u>or</u> $\cos(A)\cos(B)$ <u>or</u> $\sin(A)\sin(B)$ where $A \neq B$, use the **product-sum formulae** (given in the formula booklet).

- If asked to integrate a fraction whose numerator is not the derivative of the denominator and whose denominator is a quadratic equation that can be factorised, then you split it into **partial fractions**. Remember to check if the fraction is proper or **improper** (☞ long division).

- **Integration by Substitution:** Remember to also change the limits from *x*-values to *u*-values.

<u>Note:</u> It is always useful to remember that:

$\cos ec(x) = \frac{1}{\sin(x)}$	$\sec(x) = \frac{1}{\cos(x)}$	$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$