C4 - Chapter 1 - Partial fractions

* Any fraction with linear terms in the denominator can be split into partial fractions
* In case of repeated linear terms in the denominator, you repeat the term increasing the power every time by 1 until you match the original power of the repeated term.
* An improper algebraic fraction is one where the degree of the numerator is greater than or equal to the degree of the denominator.

* To express an algebraic fraction as a sum of partial fractions:
    - Always start by checking if the fraction is improper. If yes, then do long-division
    - Factorise the denominator if needed
    - Check for repeated linear factors in the denominator
    - Find the values of the unknown constants A, B, C etc

* Remember always to finish off by writing out your final answer

Example

Express $\dfrac{x^4}{x^2-2x+1}$ as a sum of partial fractions.

$$x^4+0x^3+0x^2+0x+0 \;\big\lfloor\, \underline{x^2-2x+1}$$

$$-\;\underline{x^4-2x^3+x^2} \qquad\qquad x^2+2x+3$$

$$2x^3-x^2+0x+0$$

$$-\;\underline{2x^3-4x^2+2x}$$

$$3x^2-2x+0$$

$$-\;\underline{3x^2-6x+3}$$

$$4x-3$$

$$\frac{x^4}{x^2-2x+1} = x^2+2x+3+\frac{4x-3}{x^2-2x+1}$$

$$\frac{4x-3}{x^2-2x+1} = \frac{4x-3}{(x-1)^2} = \frac{A}{x-1}+\frac{B}{(x-1)^2}$$

$$4x-3 = A(x-1)+B$$

$$x=1 \implies 1=B$$

$$x=0 \implies -3=-A+B$$

$$-3=-A+1 \implies A=4$$

$$\therefore \frac{x^4}{x^2-2x+1} = x^2+2x+3+\frac{4}{x-1}+\frac{1}{(x-1)^2}$$