THE GC SCHOOL OF CAREERS

DEPARTMENT OF MATHEMATICS

EXTRA PRACTICE

CORE MATHEMATICS 4

BINOMIAL EXPANSION

EXERCISES

1. Given that $f(x) \equiv \dfrac{9 - 3x - 12x^2}{(1-x)(1+2x)} \equiv A + \dfrac{B}{1-x} + \dfrac{C}{1+2x}$

   (a) find the values of the constants A, B and C.

   (b) Given that $|x| < \frac{1}{2}$, expand $f(x)$ in ascending powers of x up to and including the

   term x^3, simplifying each coefficient.

   [1993]

2. $f(x) = (1 + 3x)^{-1}$

   (a) Expand $f(x)$ in ascending powers of x up to and including the term in x^3, stating

   the range of values of x for which the expansion is valid.

   (b) Hence show that, for small x, $\dfrac{1+x}{1+3x} \approx 1 - 2x + 6x^2 - 18x^3$.

   (c) Taking a suitable value for x, which should be stated, use the series expansion in

   part(b) to find an approximate value for $\dfrac{101}{103}$, giving your answer to 5 decimal places.

   [2001]

3. **(a)** Show that if $|2x| < 1$, then $\sqrt{1 - 2x} \approx 1 - x - \dfrac{x^2}{2}$.

   (b) By substituting $x = 0.05$, find an approximation to $\sqrt{10}$ giving your answer to 3
   significant figures.

4. In the binomial expansion of $(4 + bx)^{\frac{1}{2}}$ the coefficient of x^2 is -9. Find:

   (a) the possible values of b,

   (b) the corresponding coefficient of x.

ANSWERS

1. **(a)** $A = 6, B = -2, C = 5$

   (b) $f(x) = 9 - 12x + 18x^2 - 42x^3 + ...$

2. **(a)** $f(x) = 1 - 3x + 9x^2 - 27x^3 + ...$ $|x| < \frac{1}{3}$

   (b) $1 - 2x + 6x^2 - 18x^3$

   (c) 0.98058

3. **(b)** 3.16

4. **(a)** $b = \pm 24$ **(b)** ± 6