## Chapter 4 - Extra practice 2 - Solutions

1. a) $\frac{d y}{d x}=e^{x} \sin 2 x+2 e^{x} \cos 2 x$

$$
\begin{aligned}
\left.\frac{d y}{d x}\right|_{x=0}=2 \Rightarrow \quad y-0 & =2(x-0) \\
y & =2 x
\end{aligned}
$$

b) When $x=\pi, y=\left.0 \quad \frac{d y}{d x}\right|_{x=\pi}=2 e^{\pi} \quad \Rightarrow m_{\text {NORMAL }}=-\frac{1}{2 e^{\pi}}$

$$
\begin{aligned}
\Rightarrow & y-0=\frac{-1}{2 e^{\pi}}(x-\pi) \\
\Rightarrow & 2 e^{\pi} y=-x+\pi \\
& 2 e^{\pi} y+x=\pi \quad \text { AS REQUIRED }
\end{aligned}
$$

2. a) $\frac{d x}{d t}=-2 t \quad \frac{d y}{d t}=4 \quad \Rightarrow \frac{d y}{d x}=\frac{4}{-2 t}=\frac{-2}{t}$
b) When $x=-14, y=16 \Rightarrow 16=4 t \Rightarrow t=4$

$$
\begin{array}{r}
\left.\frac{d y}{d x}\right|_{t=4}=-2 / 4 \Rightarrow m_{\text {NORMAL }}=2 \Rightarrow y-16=2(x+14) \\
y=2 x+44
\end{array}
$$

3. a) $\frac{d x}{d t}=-4 \quad \frac{d y}{d t}=-\frac{2}{t^{2}} \Rightarrow \frac{d y}{d x}=\frac{-2 / t^{2}}{-4}=\frac{1}{2 t^{2}}$
b) When $t=2, x=-5, y=2, \frac{d y}{d x}=\frac{1}{8}$

$$
\begin{gathered}
\Rightarrow \quad y-2=\frac{1}{8}(x+5) \\
\Rightarrow \quad 8 y-x-21=0
\end{gathered}
$$

c) $x=3-4 t \Rightarrow t=\frac{3-x}{4}$

$$
\begin{aligned}
\text { Substitute } \begin{aligned}
t=\frac{3-x}{4} & \text { into } y=1+\frac{2}{t} \\
\Rightarrow & y=1+\frac{2}{\frac{3-x}{4}} \\
& y=1+\frac{8}{3-x} \\
& y-1=\frac{8}{3-x} \\
& (y-1)(3-x)-8=0 \\
& (y-1)(x-3)+8=0 \text { AS REQUIRED }
\end{aligned}
\end{aligned}
$$

$6 x+8 y \frac{d y}{d x}-2+6 y+6 x \frac{d y}{d x}=0$
When $x=1, y=-2 \Rightarrow 6-16 \frac{d y}{d x}-2-12+6 \frac{d y}{d x}=0 \Rightarrow \frac{d y}{d x}=\frac{4}{5}$

$$
\begin{aligned}
& y+2=\frac{4}{5}(x-1) \\
& 5 y-4 x+14=0
\end{aligned}
$$

5. a) $14 x+48 y+48 x \frac{d y}{d x}-14 y \frac{d y}{d x}=0 \Rightarrow \frac{d y}{d x}=\frac{14 x+48 y}{14 y-48 x}$
b) $\frac{2}{11}=\frac{14 x+48 y}{14 y-48 x}$

$$
\begin{aligned}
& 28 y-96 x=154 x+528 y \\
& 0=250 x+500 y \\
& x+2 y=0 \quad \text { AS REQUIRED }
\end{aligned}
$$

c) Substitute $x=-2 y \Rightarrow 7(-2 y)^{2}+48(-2 y) y-7 y^{2}+75=0$

$$
\begin{gathered}
-75 y^{2}+75=0 \\
y^{2}=1 \\
y=1 \quad \stackrel{O R}{=} \quad \begin{array}{l}
y=-1 \\
x=-2
\end{array} \\
\begin{array}{l}
x=2 \\
(-2,1)
\end{array} \\
\\
(2,-1)
\end{gathered}
$$

