Chapter 4 - Extra practice 2 - Solutions

1. a) $\frac{dy}{dx} = e^x \sin 2x + 2e^x \cos 2x$

$\left.\frac{dy}{dx}\right|_{x=0} = 2 \quad \Rightarrow \quad y - 0 = 2(x-0)$

$y = 2x$

b) When $x = \pi$, $y = 0$ $\quad \left.\frac{dy}{dx}\right|_{x=\pi} = 2e^{\pi} \quad \Rightarrow m_{NORMAL} = \frac{-1}{2e^{\pi}}$

$\Rightarrow \quad y - 0 = \frac{-1}{2e^{\pi}}(x - \pi)$

$\Rightarrow \quad 2e^{\pi}y = -x + \pi$

$2e^{\pi}y + x = \pi$ AS REQUIRED

2. a) $\frac{dx}{dt} = -2t \quad \frac{dy}{dt} = 4 \quad \Rightarrow \frac{dy}{dx} = \frac{4}{-2t} = \frac{-2}{t}$

b) When $x = -14$, $y = 16 \Rightarrow 16 = 4t \Rightarrow t = 4$

$\left.\frac{dy}{dx}\right|_{t=4} = -2/4 \quad \Rightarrow \quad m_{NORMAL} = 2 \quad \Rightarrow \quad y - 16 = 2(x+14)$

$y = 2x + 44$

3. a) $\frac{dx}{dt} = -4 \quad \frac{dy}{dt} = \frac{-2}{t^2} \quad \Rightarrow \frac{dy}{dx} = \frac{-2/t^2}{-4} = \frac{1}{2t^2}$

b) When $t = 2$, $x = -5$, $y = 2$, $\frac{dy}{dx} = \frac{1}{8}$

$\Rightarrow \quad y - 2 = \frac{1}{8}(x+5)$

$\Rightarrow \quad 8y - x - 21 = 0$

c) $x = 3 - 4t \quad \Rightarrow \quad t = \frac{3-x}{4}$

Substitute $t = \frac{3-x}{4}$ into $y = 1 + \frac{2}{t}$

$\Rightarrow \quad y = 1 + \frac{2}{\frac{3-x}{4}}$

$y = 1 + \frac{8}{3-x}$

$y - 1 = \frac{8}{3-x}$

$(y-1)(3-x) - 8 = 0$

$(y-1)(x-3) + 8 = 0$ AS REQUIRED

4. $6x + 8y\frac{dy}{dx} - 2 + 6y + 6x\frac{dy}{dx} = 0$

When $x=1, y=-2 \Rightarrow 6 - 16\frac{dy}{dx} - 2 - 12 + 6\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{4}{5}$

$$y+2 = \frac{4}{5}(x-1)$$

$$5y - 4x + 14 = 0$$

5. a) $14x + 48y + 48x\frac{dy}{dx} - 14y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{14x+48y}{14y-48x}$

b) $\frac{2}{11} = \frac{14x+48y}{14y-48x}$

$28y - 96x = 154x + 528y$

$0 = 250x + 500y$

$x + 2y = 0$ AS REQUIRED

c) Substitute $x = -2y \Rightarrow 7(-2y)^2 + 48(-2y)y - 7y^2 + 75 = 0$

$$-75y^2 + 75 = 0$$

$$y^2 = 1$$

$y = 1$ OR $y = -1$

$x = -2$ $x = 2$

$(-2, 1)$ $(2, -1)$