THE GC SCHOOL OF CAREERS

DEPARTMENT OF MATHEMATICS

EXTRA PRACTICE

CORE MATHEMATICS 4

DIFFERENTIATION

EXERCISES

1. The curve C is given by the equations $y = 2t$, $x = t^2 + t^3$ where t is a parameter.
Find the equation of the normal to C at the point P on C where $t = -2$.

2. At time t seconds the surface area of a cube is A cm^2 and its volume is V cm^3.
The volume of the cube is expanding at a uniform rate of 2 cm^{3}s^{-1}.

Show that $\dfrac{dA}{dt} = kA^{-\frac{1}{2}}$, where k is a constant to be determined.

3. Find $\dfrac{dy}{dx}$ when:

(a) $y = 2^x$

(b) $y = x \cdot 3^x$

(c) $y = 4^{\sqrt{x}}$

(d) $x^2 + 3y^2 - 6x = 12$

(e) $x^4 - 4x^2y^2 + y = 8$

4. The curve C has parametric equations $x = 4\cos 2t$, $y = 3\sin t$, $-\dfrac{\pi}{2} < t < \dfrac{\pi}{2}$.

A is the point $\left(2, 1\dfrac{1}{2}\right)$ and lis on C.

(a) Find the value of t at the point A.

(b) Find $\dfrac{dy}{dx}$ in terms of t.

(c) Show that an equation of the normal to C at A is $6y - 16x + 23 = 0$.

The normal at A cuts C at the point B.

(d) Find the y-coordinate of the point B.

[1997]

5. The curve C has equation $5x^2 + 2xy - 3y^2 + 3 = 0$

The point P on the curve C has coordinates (1, 2).

(a) Find the gradient of the curve at P.

(b) Find the equation of the normal to the curve C at P, in the form $y = ax + b$, where a and b are constants.

[2004]

ANSWERS

1. $y+4x+20=0$

2. $\frac{dA}{dt}=8\sqrt{6}A^{-\frac{1}{2}}$**,** $k=8\sqrt{6}$

3. **(a)** $2^x \ln 2$ **(b)** $3^x + x\cdot 3^x \ln 3$ **(c)** $\frac{1}{2}x^{-\frac{1}{2}}4^{\sqrt{x}}\ln 4$ **(d)** $\frac{3-x}{3y}$ **(e)** $\frac{-4x(x^2+2y^2)}{8x^2y+1}$

4. **(a)** $t=\frac{\pi}{6}$ **(b)** $\frac{dy}{dx}=-\frac{3}{16\sin t}$ **(c)** $6y-16x+23=0$ **(d)** $-\frac{123}{64}$

5. **(a)** $\frac{dy}{dx}=\frac{7}{5}$ **(b)** $y=-\frac{5}{7}x+\frac{19}{7}$