Chapter 4 - Extra practice - Solutions

1. i) $6x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{3x}{y}$

ii) $2xy + x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{2-2xy}{x^2+3y^2}$

iii) $6x - 2y\frac{dy}{dx} + 5 - 6\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{6x+5}{2y+6}$

iv) $3y^2\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} - 2 = 0 \Rightarrow \frac{dy}{dx} = \frac{2-2xy}{3y^2+x^2}$

v) $3y^2\frac{dy}{dx} + \ln y + \frac{x}{y}\frac{dy}{dx} = 6x \Rightarrow \frac{dy}{dx} = \frac{6x-\ln y}{3y^2+\frac{x}{y}}$

2. $\cos y\frac{dy}{dx} = y + x\frac{dy}{dx} + 2x$

$\frac{dy}{dx} = \frac{y+2x}{\cos y - x}$

3. $8x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

Substitute $x=1, y=2 \Rightarrow 8+4+2\frac{dy}{dx} + 4\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -2$

4. $4x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

Stationary points $\Rightarrow \frac{dy}{dx} = 0$

$4x + y + x\frac{dy}{dx}^{\to 0} + 2y\frac{dy}{dx}^{\to 0} = 0$

$4x + y = 0 \Rightarrow y = -4x$

Substitute $y = -4x$ into the original equation

$2x^2 + x(-4x) + (-4x)^2 = 14$

$14x^2 = 14$

$x = \pm 1$

$x = 1 \Rightarrow y = -4$ $\therefore (1, -4)$

$x = -1 \Rightarrow y = 4$ $(-1, 4)$

5. $3x^2 + 8xy + 4x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

Substitute $x=1, y=1 \Rightarrow 3 + 8 + 4\frac{dy}{dx} + 3\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -11/7$

$\Rightarrow m_{NORMAL} = 7/11$

$\therefore \quad y-1 = 7/11\,(x-1)$

$11y - 11 = 7x - 7$

$11y - 7x - 4 = 0$

6. a) $2xy + x^2\frac{dy}{dx} - y^2 - 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y^2-2xy}{x^2-2xy}$ AS REQUIRED

b) $\frac{dy}{dx} = 0 \Rightarrow y^2 - 2xy = 0$

$\Rightarrow y(y-2x) = 0$

$y = 0$ Reject * OR $y = 2x$ AS REQUIRED

* since substituting $y=0$ into the equation of the curve leads to $0=2$.

c) Tangent parallel to the x-axis $\Rightarrow \frac{dy}{dx} = 0$ (flat line)

$\Rightarrow y = 2x$ Substitute $y = 2x$

$\Rightarrow x^2(2x) - x(2x)^2 = 2$

$-2x^3 = 2$

$x^3 = -1$

$x = -1, y = -2 \Rightarrow (-1, -2)$