The points A and B have position vectors $(5\mathbf{i} + 8\mathbf{j} - 4\mathbf{k})$ and $(8\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$ respectively.

(a) Find a vector equation for the line l which passes through A and B.

(b) Given that the point with coordinates $(p, 4p, q)$ lies on l, find the value of p and the value of q.

The lines l_1 and l_2 have vector equations

$$\text{and } \mathbf{r} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix}$$

$$\text{and } \mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ 9 \end{pmatrix} + s\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix} \text{ respectively.}$$

(a) Show that l_1 and l_2 intersect.

(b) Find the coordinates of their point of intersection.

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place.

The points A and B have position vectors $(3\mathbf{i} + t\mathbf{j} + 5\mathbf{k})$ and $(7\mathbf{i} + \mathbf{j} + t\mathbf{k})$ respectively.

(a) Find $|\overrightarrow{AB}|$ in terms of t.

(b) Find the value of t that makes $|\overrightarrow{AB}|$ a minimum.

(c) Find the minimum value of $|\overrightarrow{AB}|$.

The equations of the lines l_1 and l_2 are given by

$$l_1: \quad \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$
$$l_2: \quad \mathbf{r} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$$

where λ and μ are parameters.

(a) Show that l_1 and l_2 intersect and find the coordinates of Q, their point of intersection.

(b) Show that l_1 is perpendicular to l_2.

The point P with x-coordinate 3 lies on the line l_1 and the point R with x-coordinate 4 lies on the line l_2.

(c) Find, in its simplest form, the exact area of the triangle PQR.

Relative to a fixed origin O, the point A has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, the point B has position vector $5\mathbf{i} + \mathbf{j} + \mathbf{k}$, and the point C has position vector $7\mathbf{i} - \mathbf{j}$.

(a) Find the cosine of angle ABC.

(b) Find the exact value of the area of triangle ABC.

The point D has position vector $7\mathbf{i} + 3\mathbf{k}$.

(c) Show that AC is perpendicular to CD.

(d) Find the ratio $AD : DB$.

Relative to a fixed origin O, the point A has position vector $4\mathbf{i} + 8\mathbf{j} - \mathbf{k}$, and the point B has position vector $7\mathbf{i} + 14\mathbf{j} + 5\mathbf{k}$.

(a) Find the vector $\overrightarrow{AB}$.

(b) Calculate the cosine of $\angle OAB$.

(c) Show that, for all values of λ, the point P with position vector $\lambda\mathbf{i} + 2\lambda\mathbf{j} + (2\lambda - 9)\mathbf{k}$ lies on the line through A and B.

(d) Find the value of λ for which OP is perpendicular to AB.

(e) Hence find the coordinates of the foot of the perpendicular from O to AB.