FP1 - Chapter 1 - Complex numbers - Summary

* $\sqrt{-1} = i \iff i^2 = -1$

* A complex number is written in the form $a+bi$ where a and b are real numbers

$$\underbrace{a}_{\text{real part}} + \underbrace{b}_{\text{imaginary part}} i$$

* If $z = a+bi$, then the complex conjugate of z, written as z^* is $z^* = a-bi$. z and z^* are known as a complex conjugate pair.

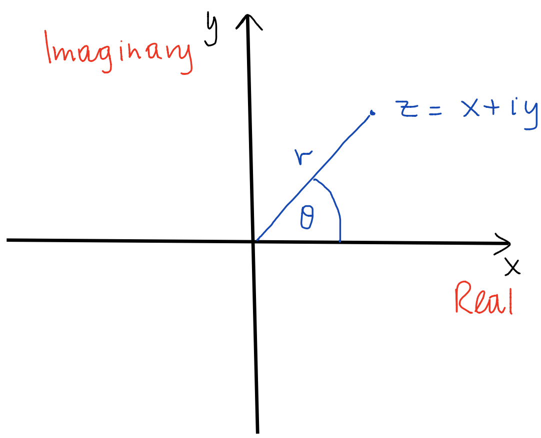
* You can divide two complex numbers by using the complex conjugate pair of the denominator

eg $\dfrac{10+5i}{1+2i} = \dfrac{10+5i}{1+2i} \cdot \dfrac{1-2i}{1-2i} = \dfrac{10-20i+5i-10i^2}{1-\cancel{2i}+\cancel{2i}-4i^2} = \dfrac{10-15i+10}{1+4}$

$= \dfrac{20-15i}{5} = 4-3i$

Note that multiplication by the conjugate pair of the denominator results in a real denominator, ie $zz^* =$ REAL

* Complex numbers may be represented on an Argand diagram



* The modulus of $z = x+iy$ is $|z| = x^2+y^2$

* The argument of $z = x+iy$ is the angle θ between the positive x-axis and the vector representing z on the Argand diagram

$\tan\theta = y/x \qquad -\pi < \theta \leq \pi$

* The modulus argument form of $z = x+iy$ is

$z = r(\cos\theta + i\sin\theta)$ where $r>0$ and $-\pi < \theta \leq \pi$

* $|z_1 z_2| = |z_1| \cdot |z_2|$

* For any given equation, complex roots occur as complex-conjugate pairs ie if z is a root of $f(x)$ then z^* is also a root.

  It follows that:

  a) A quadratic will have either two complex roots or all roots real

  b) A cubic will either have one real root and two complex (which will form a complex conjugate pair) or three real roots

  c) A quartic equation will either have 4 real roots, 2 real and 2 complex that form a complex conjugate pair, or 4 complex roots (2 pairs of complex conjugate pairs).

  You can use the above to find all the roots of a given equation.

* You can solve problems by equating the real and imaginary parts of complex numbers.

  i.e. if $x_1 + y_1 i = x_2 + y_2 i$

  then $x_1 = x_2$ and $y_1 = y_2$