FP1 - Chapter 2 - Numerical solutions of equations - Summary

* If there is an interval say (a,b) in which $f(x)$ changes sign, then a root of the equation $f(x)=0$ lies between a and b.

* Interval bisection

eg. Use interval bisection to find the positive root of $\sqrt{11}$ to 1 decimal place.

We need to find x such that $x=\sqrt{11}$ $\Rightarrow x^2=11$
$\Rightarrow x^2-11=0$

Hence, consider $f(x)=x^2-11$

a	$f(a)$	b	$f(b)$	$\frac{a+b}{2}$	$f\left(\frac{a+b}{2}\right)$
3	-2	4	5	3.5	1.25
3	-2	3.5	1.25	3.25	-0.4375
3.25	-0.4375	3.5	1.25	3.375	0.390625
3.25	-0.4375	3.375	0.390625	3.3125	-0.0273437
3.3125	-0.0273437	3.375	0.390625	3.34375	0.180664

Hence, $\sqrt{11}=3.3$ to 1 dp

Always draw up a table similar to the above. Once a row is completed choose a or b such that there is a change of sign with the midpoint. The chosen value and the midpoint will be your new a and b to start a new row.
So, every new row represents essentially a tighter interval over which a change of sign is observed.

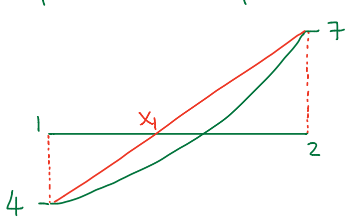
* Linear interpolation

For this method you draw a sketch of $f(x)$ for a given interval and then use similar triangles to find the next approximation.

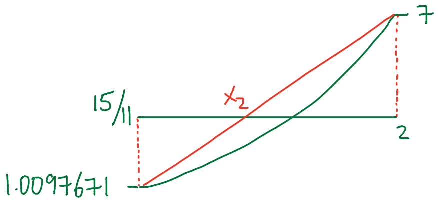
eg. $f(x)=x^3+4x-9$ has a root in the interval $[1,2]$.

Use linear interpolation twice to find an approximation to this root.

$f(1)=-4$ $\quad f(2)=7$



$$\frac{x_1-1}{4}=\frac{2-x_1}{7}$$
$$7x_1-7=8-4x_1$$
$$11x_1=15$$
$$x_1=15/11$$
$$f(15/11)=-1.0097671\ldots$$



$$\frac{x_2 - 15/11}{1.0097671} = \frac{2 - x_2}{7}$$

$$7x_2 - 7(15/11) = 2(1.0097671) - 1.0097671x_2$$

$$x_2 = 1.4438607$$

* Newton-Raphson process

Use the formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ to generate the next approximation.

Note that the Newton-Raphson process may not always give a better approximation and may take you further away from the root.