FP1 - Chapter 3 - Coordinate systems - Summary

＊ A Cartesian equation is one involving x and y only

＊ A curve may be described parametrically if x and y are expressed in terms of a third variable, usually θ or t. This third variable is known as the parameter.
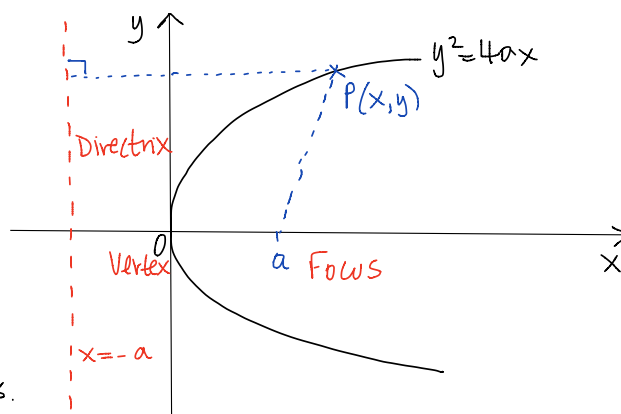
＊ Parabola

- Cartesian equation: $y^2 = 4ax$
  where a is a positive constant

- Parametric equations

$$x = at^2 \qquad y = 2at$$
where a is a positive constant

- Curve is symmetric about the x-axis.

- The focus, S, has coordinates $(a, 0)$

- The directrix has equation $x = -a$

- The vertex is at the point $(0,0)$.

A parabola is the locus of points where every point on the parabola is the same distance from the focus, S and the directrix

$$\Rightarrow \quad \underbrace{\sqrt{(x-a)^2 + (y-0)^2}}_{\text{Distance from focus}} = \underbrace{x + a}_{\text{Distance from directrix}}$$

$$\sqrt{x^2 - 2ax + a^2 + y^2} = x + a$$

$$x^2 - 2ax + a^2 + y^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$

$$y^2 = 4ax$$

* Rectangular hyperbola

- Cartesian equation: $xy = c^2$, where c is a positive constant

- Parametric equations: $x = ct$ $y = \dfrac{c}{t}$, $t \neq 0$

- The curves has asymptotes $x = 0$ and $y = 0$.