FP1 - Chapter 4 - Matrix algebra - Summary

* A matrix is an array of numbers
* An $n \times m$ matrix has n rows and m columns
* $A \pm B$ can only be calculated if A and B have the same dimensions
  In this case you simply add or subtract the corresponding entries
* kA, where k is a scalar (number), can be found by multiplying each entry of A by k.
* $A \times B$ can be calculated only if the number of columns of A is equal to the number of rows of B

  If $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$

  then $A \times B = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{21}b_{22} \end{pmatrix}$

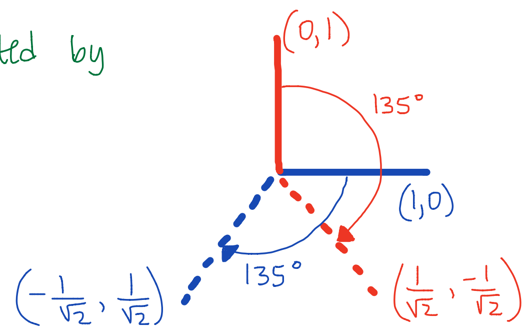
* Note that in general $AB \neq BA$
* A linear transformation: ① Involves only linear expressions of x and y
  ② does not change the position of the origin.
* Matrices can be used to represent linear transformations
  The transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$ is represented by $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$
* In general, the image of the point (x, y) under the transformation M is given by $M\begin{pmatrix} x \\ y \end{pmatrix}$
* You should be able to identify matrices representing the following transformations
  - Rotation about $(0,0)$ of angles that are multiples of $45°$
  - Enlargement with centre $(0,0)$, scale factor k.

- Reflection along the axes or the lines $y = \pm x$.

* To identify the transformation represented by a matrix, consider the two columns of the matrix. Each of these corresponds to the images of $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, respectively.

eg. Identify the transformation represented by

$$M = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$



Hence, M represents a rotation about (0,0) through 135° clockwise.

* If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

- $AA^{-1} = A^{-1}A = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

- $\det A = ad - bc$

* Note that A^{-1} exists only if $\det(A) \neq 0$. If $\det(A) = 0$ then A is known as a singular matrix.

* If A and B are non-singular matrices then $(AB)^{-1} = B^{-1}A^{-1}$

* Area of image = Area of object $\times$ $|\det(M)|$ where M is the matrix representing the transformation applied

* Matrices can be used to solve simultaneous equations

eg. $\begin{aligned} 4x - y &= 11 \\ 3x + 2y &= 0 \end{aligned} \Rightarrow \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$

Consider $A = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$. Then $A^{-1} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$

Hence, $\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 11 \\ 0 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 11 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

ie $x = 2$, $y = -3$