June 05 Q4. Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,\mathrm{d}x.$$

(7)

Jan 06 Q3. Using the substitution $u^2 = 2x - 1$, or otherwise, find the exact value of

$$\int_1^5 \frac{3x}{\sqrt{(2x-1)}}\,\mathrm{d}x.$$

(8)

Jan 07 Q8.

$$I = \int_0^5 \mathrm{e}^{\sqrt{(3x+1)}}\,\mathrm{d}x.$$

(*a*) Given that $y = \mathrm{e}^{\sqrt{(3x+1)}}$, copy and complete the table with the values of y corresponding to $x = 2$, 3 and 4.

x	0	1	2	3	4	5
y	e^1	e^2				e^4

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the original integral I, giving your answer to 4 significant figures.

(3)

(*c*) Use the substitution $t = \sqrt{(3x + 1)}$ to show that I may be expressed as $\int_a^b kt\mathrm{e}^t\,\mathrm{d}t$, giving the values of a, b and k.

(5)

(*d*) Use integration by parts to evaluate this integral, and hence find the value of I correct to 4 significant figures, showing all the steps in your working.

(5)

June 07 Q2. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\,\mathrm{d}x.$$

(6)

Jan 10 Q8. (*a*) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_{1}^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,\mathrm{d}x\,.$$

(7)

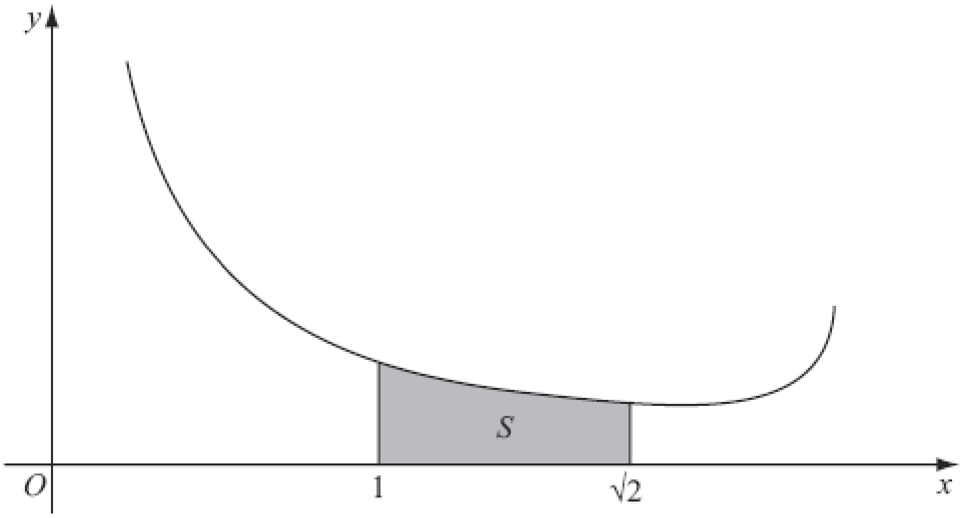

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S, shown in Figure 3, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Using your answer to part (*a*), find the exact volume of the solid of revolution formed.

(3)

June 10 Q2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_{0}^{\frac{\pi}{2}} \mathrm{e}^{\cos x+1}\sin x\ \ \mathrm{d}x = \mathrm{e}(\mathrm{e}-1).$$

(6)

Jan 11 Q7.

$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,dx.$$

(a) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(b) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(c) Using the substitution $x = (u - 4)^2 + 1$, or otherwise, and integrating, find the exact value of I.

(8)

June 11 Q4.

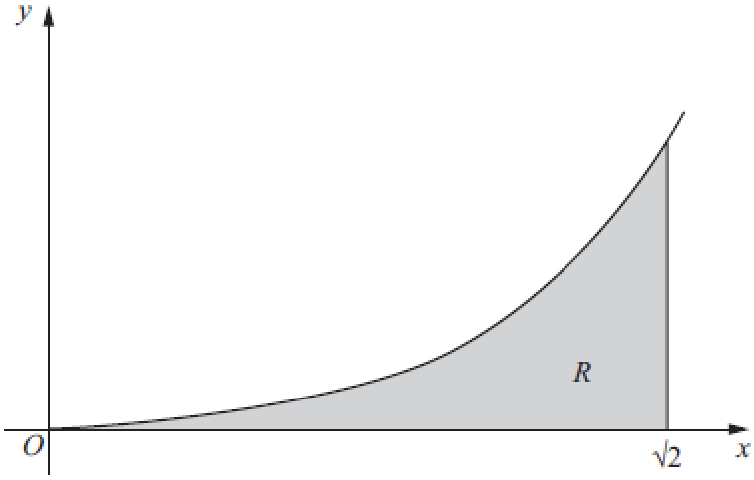

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln (x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln (x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(*a*) Complete the table above giving the missing values of y to 4 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(*c*) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_{2}^{4} (u-2)\ln u \; du .$$

(4)

(*d*) Hence, or otherwise, find the exact area of R.

(6)

Jan 12 Q6.

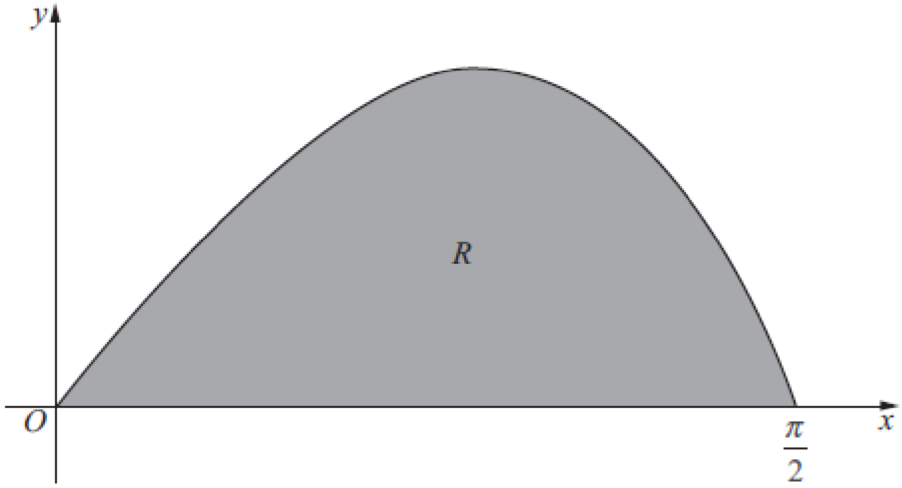

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \frac{2\sin 2x}{(1+\cos x)}$, $0 \leq x \leq \frac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \frac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1 + \cos x) - 4\cos x + k,$$

where k is a constant.

(5)

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures.

(3)