C2 - MAY 2012

1 $(2-3x)^5 = 2^5\left(1-\frac{3x}{2}\right)^5 = 32\left\{1+5\left(-\frac{3x}{2}\right)+\frac{5.4}{2!}\left(-\frac{3x}{2}\right)^2+\dots\right\}$

$= 32-240x+720x^2+\dots$

2. $\log_{10} 100 = 2 \qquad 10^2 = 100$

$2\log_3 x - \log_3(x-2) = 2$

$\log_3 x^2 - \log_3(x-2) = 2 \rightarrow \log_3\left(\frac{x^2}{x-2}\right) = 2$

$3^2 = \frac{x^2}{x-2}$

$9(x-2) = x^2$

$9x-18 = x^2$

$0 = x^2-9x+18$

$0 = (x-3)(x-6)$

$x=3$ OR $x=6$

3. $(x-a)^2+(y-b)^2=r^2$ Circle centre (a,b) radius r

a) $x^2+y^2-20x-16y+139=0$

$(x-10)^2-100+(y-8)^2-64+139=0$

$(x-10)^2+(y-8)^2=25 \qquad (10,8)$

b) $r^2 = 25 \Rightarrow r = 5$ AS REQUIRED

c) Substitute $x=13$

$$(13-10)^2 + (y-8)^2 = 25$$
$$(y-8)^2 = 16$$
$$y-8 = \pm 4$$
$$y = 4+8 = 12 \quad \underline{\underline{OR}} \quad y = -4+8 = 4$$

d) Perimeter $= r + r +$ length of arc

$$= 5 + 5 + 5 \cdot 1.855$$
$$= 19.275$$

4. If $x-\alpha$ is a factor of $f(x)$ then $f(\alpha)=0$

a) $f(-2) = 2(-2)^3 - 7(-2)^2 - 10(-2) + 24 = 0$

$\therefore$ $x+2$ is a factor of $f(x)$

b)
$$\begin{array}{rl}
2x^3 - 7x^2 - 10x + 24 & \underline{|\, x+2} \\
-\;\underline{2x^3 + 4x^2} \qquad\qquad & 2x^2 - 11x + 12 \\
-11x^2 - 10x + 24 & \\
-\;\underline{-11x^2 - 22x} \qquad & \\
12x + 24 & \\
-\;\underline{12x + 24} & \\
0 &
\end{array}$$

$$\begin{array}{c|c} 7 & 3 \\ \hline 1 & 2 \end{array} \qquad \frac{7}{3} = 2 + \frac{1}{3}$$

$$2x^3 - 7x^2 - 10x + 24 = (x+2)(2x^2 - 11x + 12) = (x+2)(2x-3)(x-4)$$

5 a) $y = 10 - x \qquad y = 10x - x^2 - 8$

$$10 - x = 10x - x^2 - 8$$
$$x^2 - 11x + 18 = 0$$
$$(x-2)(x-9) = 0$$
$$x = 2 \quad \underline{\underline{OR}} \quad x = 9$$
$$y = 8 \qquad y = 1 \qquad (2,8) \qquad (9,1)$$

b) $R = \int_2^9 10x - x^2 - 8 - (10 - x)\,dx = \int_2^9 11x - x^2 - 18\,dx$

$$= \left[\frac{11x^2}{2} - \frac{x^3}{3} - 18x\right]_2^9 = 343/6$$

7. a) $x = 0.5 \quad y = 1.494$

$x = 0.75 \quad y = 1.741$

b) $\int_0^1 \sqrt{3^x + x}\,dx \approx \frac{1}{2} \cdot 0.25\{1 + 2(1.251 + 1.494 + 1.741) + 2\}$

$$= 1.4965$$

Height of each trapezium = Difference between two consecutive values of x

8. a) $V = \pi r^2 h$

$60 = \pi x^2 h$

$h = \dfrac{60}{\pi x^2}$

b) $A = 2\pi x^2 + 2\pi x h$

$= 2\pi x^2 + 2\pi x \cdot \dfrac{60}{\pi x^2}$

$= 2\pi x^2 + \dfrac{120}{x}$

c) $A = 2\pi x^2 + 120x^{-1}$ $\quad \dfrac{dA}{dx} = 4\pi x - 120x^{-2} = 0$

$4\pi x = \dfrac{120}{x^2} \Rightarrow x^3 = \dfrac{120}{4\pi} \quad x = \sqrt[3]{\dfrac{120}{4\pi}} = 2.12$

d) When $x = \sqrt[3]{\dfrac{120}{4\pi}}$ $\quad A = 85$ (to the nearest integer)

e) $\dfrac{d^2A}{dx^2} = 4\pi + 240x^{-3}$

$\left.\dfrac{d^2A}{dx^2}\right|_{x=\sqrt[3]{\frac{120}{4\pi}}} = 37.7 \boxed{> 0}$ $\therefore$ Minimum

Necessary to get the point

C3 - JUNE 2010

1 a) LHS $= \dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos^2\theta} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta =$ RHS

b) $\dfrac{2\sin 2\theta}{1+\cos 2\theta} = 1 \qquad \dfrac{\sin 2\theta}{1+\cos 2\theta} = \dfrac{1}{2}$

$\tan\theta = 1/2$

$\alpha = 26.565$

$\theta = 180n + 26.565$

$\theta = -153.4, 26.6$

2. $y = 3(5-3x)^{-2}$

$\dfrac{dy}{dx} = 3(-2)(-3)(5-3x)^{-3} = \dfrac{18}{(5-3x)^3}$

$\left.\dfrac{dy}{dx}\right|_{x=2} = \dfrac{18}{(5-3\cdot 2)^3} = -18 \qquad m_{\text{NORMAL}} = \dfrac{1}{18}$

When $x=2$, $y = \dfrac{3}{(5-3\cdot 2)^2} = 3$

$y - y_1 = m(x - x_1) \qquad y - 3 = \dfrac{1}{18}(x-2)$

$18y - x - 52 = 0$

3. a) $f(1.2) = 0.492 > 0$ Change of sign therefore
$f(1.3) = -0.0487 < 0$ a root exists in the interval $(1.2, 1.3)$

b) $4\operatorname{cosec}x - 4x + 1 = 0$

$$\frac{4}{\sin x} + 1 = 4x$$

$$x = \frac{4}{4\sin x} + \frac{1}{4} = \frac{1}{\sin x} + \frac{1}{4}$$ AS REQUIRED

c) $x_1 = 1.3038$ $x_2 = 1.2867$ $x_3 = 1.2917$

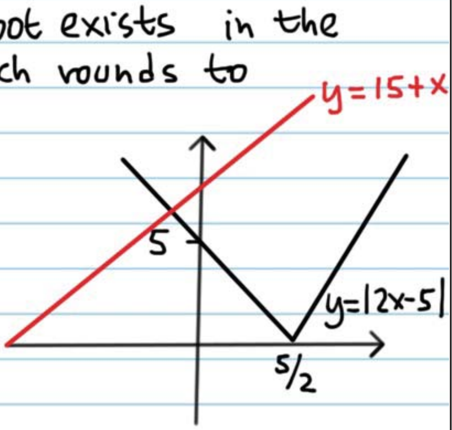
d) $f(1.2905) = 0.00045 > 0$
$f(1.2915) = -0.0048 < 0$

Change of sign, therefore a root exists in the interval $(1.2905, 1.2915)$ which rounds to 1.291 correct to 3dp's

4 a) $f(x) = |2x-5|$

$f(0) = |0-5| = 5$

$f(x) = 0 \Rightarrow 2x-5=0$
$x = 5/2$



b) $f(x) = 15 + x$

$2x - 5 = 15 + x$

$x = 20$

$-(2x-5) = 15 + x$

$-2x + 5 = 15 + x$

$-10 = 3x$

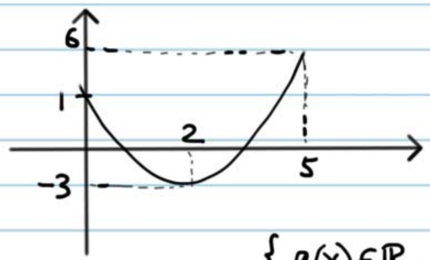
$x = -10/3$

c) $fg(2) = f[g(2)] = f(-3) = |2(-3) - 5| = 11$

d) $g(x) = x^2 - 4x + 1$

Symmetry $x = \frac{-b}{2a}$

$x = \frac{4}{2} = 2$



$\{ g(x) \in \mathbb{R},\ -3 \leq g(x) \leq 6 \}$