1 a) $U_n = ar^{n-1}$ $U_{20} = 360 \left(\frac{7}{8}\right)^{19} = 28.5$

b) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{360\left(1-\left(\frac{7}{8}\right)^{20}\right)}{1-\frac{7}{8}} = 2680$

c) $S_\infty = \dfrac{a}{1-r} = \dfrac{360}{1-\frac{7}{8}} = 2880$

2. Radius $= \sqrt{(-1-0)^2 + (7-0)^2} = \sqrt{50}$

$(x-\alpha)^2 + (y-b)^2 = r^2$

$(x+1)^2 + (y-7)^2 = 50$

3. a) $\left(1+\frac{x}{4}\right)^8 = 1 + 8\left(\frac{x}{4}\right) + \dfrac{8\cdot7}{2!}\left(\frac{x}{4}\right)^2 + \dfrac{8\cdot7\cdot6}{3!}\left(\frac{x}{4}\right)^3 + \dots$

$= 1 + 2x + \dfrac{7x^2}{4} + \dfrac{7x^3}{8} + \dots$

b) $1 + \frac{x}{4} = 1.025 \Rightarrow x = 0.1 \Rightarrow 1 + 2(0.1) + \dfrac{7(0.1)^2}{4} + \dfrac{7(0.1)^3}{8}$

$= 1.2184$

4 a) $y = 3x^2$

$\log_3 y = \log_3 3x^2$

$\log_3 y = \log_3 3 + \log_3 x^2$

$\log_3 y = 1 + 2\log_3 x$ AS REQUIRED

b) $1 + 2\log_3 x = \log_3 (28x - 9)$

$\log_3 y = \log_3 (28x - 9)$

$y = 28x - 9$ $y = 3x^2$

$3x^2 = 28x - 9$
$3x^2 - 28x + 9 = 0$
$(3x - 1)(x - 9) = 0$
$x = \frac{1}{3}$ OR $x = 9$

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5 a) $f(x) = x^3 + ax^2 + bx + 3$

$f(-2) = (-2)^3 + a(-2)^2 + b(-2) + 3 = -5 + 4a - 2b = 7$

$4a - 2b = 12$

$2a - b = 6$

b) $f(1) = 1^3 + a \cdot 1^2 + b \cdot 1 + 3 = 4 + a + b = 4$

$a + b = 0$

$3a = 6$

$a = 2$

$b = -2$

6 a) When $x=2$, $y=4$ When $x=2.5$, $y=2.31$

b) $R \approx \frac{1}{2} \, 0.5 \left\{ 16.5 + 2(7.361 + 4 + 2.31 + 1.278 + 0.556) + 0 \right\}$

      $= 11.88$

c) $R = \int_1^4 16x^{-2} - \frac{x}{2} + 1 \, dx = \left[\frac{16x^{-1}}{-1} - \frac{x^2}{4} + x \right]_1^4$

                           $= \left(-4 - 4 + 4 \right) - \left(-16 - \frac{1}{4} + 1 \right)$

                           $= 45/4$

7a) $BC = 6 \times 0.95 = 5.7 \, cm$

b) Area of $ABC = \frac{1}{2} r^2 \theta = \frac{1}{2} \, 6^2 \cdot 0.95 = 17.1 \, cm^2$

c) $A\hat{D}B = \pi - 2(0.95) = 1.241592654$

    $\dfrac{6}{\sin(1.24\ldots)} = \dfrac{AD}{\sin(0.95)}$ $\Rightarrow AD = 5.16 \, cm$

                                        AS REQUIRED

d) Perimeter $= 5.16 + 5.7 + (6 - 5.16) = 11.7 \, cm$

e) R = Area of sector − Area of triangle

$$= 17.1 - \frac{1}{2} \cdot 6 \cdot 5.16 \sin(095) = 45 cm^2$$

8 a) $4 = xy + xy + \frac{\pi x^2}{4}$

$$4 = 2xy + \frac{\pi x^2}{4}$$

$$16 = 8xy + \pi x^2$$

$$y = \frac{16 - \pi x^2}{8x} \quad \text{AS REQUIRED}$$

b) $P = y + x + y + \frac{2\pi x}{4} + y + x + y$

$$= 4y + 2x + \frac{2\pi x}{4} = 4\left(\frac{16 - \pi x^2}{8x}\right) + 2x + \frac{\pi x}{2}$$

$$= \frac{64}{8x} - \frac{4\pi x^2}{8x} + 2x + \frac{\pi x}{2} = \frac{8}{x} - \frac{\pi x}{2} + 2x + \frac{\pi x}{2}$$

$$= \frac{8}{x} + 2x \quad \text{AS REQUIRED}$$

c) $\frac{dP}{dx} = -8x^{-2} + 2$ Minimum $\Rightarrow \frac{dP}{dx} = 0 \Rightarrow \frac{-8}{x^2} + 2 = 0$

$$2 = \frac{8}{x^2} \Rightarrow x^2 = 4 \Rightarrow x = 2$$

When $x = 2$, $P = 8$

d) $y = \dfrac{16 - \pi \cdot 2^2}{8 \cdot 2} = 21\,cm$

9₁) $\sin(3x - 15) = \frac{1}{2}$

$\qquad \alpha = 30$

$3x - 15 = 360n + 30 \qquad\qquad 3x - 15 = 360n + 180 - 30$

$\qquad x = 120n + 15 \qquad\qquad\qquad x = 120n + 55$

$\qquad\qquad x = 15, 55, 135, 175$

ii) $\sin(ax - b) = 0$

$\qquad \alpha = 0$

$ax - b = 2\pi n \qquad \Bigg\} \; ax - b = 0, \pi, 2\pi$

$ax - b = 2\pi n + \pi$

$ax - b = 0, \pi, 2\pi$

$x = \dfrac{b}{a}, \; \dfrac{\pi + b}{a}, \; \dfrac{2\pi + b}{a}$

$x = \dfrac{\pi}{10}, \; \dfrac{3\pi}{5}, \; \dfrac{11\pi}{10}$

$\dfrac{b}{a} = \dfrac{\pi}{10} \quad \Rightarrow 10b = \pi a \qquad\qquad \dfrac{\pi + b}{a} = \dfrac{3\pi}{5}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 5\pi + 5b = 3\pi a$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 5\pi + 5b = 3(10b)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 5\pi = 25b$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad b = \dfrac{\pi}{5} \qquad a = 2$