S1 - Chapter 8 - Discrete random variables - Summary

* A variable is a quantity that can take on any of a specified set of values.
* When the variable represents the outcome of an experiment then it is called a random variable.
* The list of all possible outcomes of an experiment is called the sample space.
* $p(x)$ and $P(X=x)$ refer to the probability of X being equal to the particular value x.
* A continuous random variable can take any value on a continuous scale (eg. weight)
* A discrete random variable can only take certain values (eg result when rolling a dice)
* A discrete random variable can be specified by
  - either its probability distribution which is a table of all possible outcomes and the corresponding probabilities for each outcome
  - or by using a probability function

eg A dice is rolled and the result is recorded

Let X = result ALWAYS DEFINE WHAT YOUR RANDOM VARIABLE REPRESENTS

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Probability distribution

$$P(X=x) = \begin{cases} \frac{1}{6} & x=1,2,3,4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

Probability function

* It follows that $\sum P(X=x) = 1$
* $F(x) = P(X \leq x)$ This is the cumulative distribution function of X

Returning to the example above

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$F(x)=P(X\leq x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$

* It follows that:

  ① The last value of $F(x)$ will always be 1, as this includes all possible outcomes

  ② The first value for $P(X=x)$ and $F(x)$ will be identical.

* $E(X) = \Sigma x P(X=x)$ This is known as the expected value of X (or mean).

* $E(X^2) = \Sigma x^2 P(X=x)$, and in general $E(X^n) = \Sigma x^n P(X=x)$

* $Var(X) = E(X^2) - [E(X)]^2$ This is the variance of X (or σ^2)

* $E(aX+b) = aE(X) + b$, where a and b are constants

* $Var(aX+b) = a^2 Var(X)$, where a and b are constants

* A discrete random variable, X, where all values are equally likely to occur is said to follow a discrete uniform distribution

  ie If X is defined over a set of n distinct values, $P(X=x) = \frac{1}{n}$ for each value of x.

* If a random variable, X, follows a discrete uniform distribution and takes values over $\{1, 2, 3, \ldots, n\}$ then

$$E(X) = \frac{n+1}{2} \qquad Var(X) = \frac{(n+1)(n-1)}{12}$$